MELUHA INTERNATIONAL SCHOOL

HYDERABAD

SR MPC JEE MAINS | **UNIT – IV ASSIGNMENT – 2** | **Date: 03-05-2020**

Time: | | **Max. Marks:**

MATHS

Syllabus: ***CALCULUS:–*** ***1. LIMITS, 2. CONTINUITY & DIFFERENTIABILITY, 3. DERIVATIVES, 4. APPLICATIONS OF DERIVATIVES , 5.INDEFINITE INTEGRATION, 6. DEFINITE INTEGRATION , 7.AREAS, 8. DIFFERENTIAL EQUATIONS***

1. Find Value of 'c' by using Rolle's theorem for $f(x)=\log(x^2+2)-\log 3$ on $[-1,1]$

(A) 0 (B) 1 (C) –1 (D) does not exists

2. The chord joining the points where x = p and x = q on the curve $y=ax^2+bx+c$ is parallel to the tangent at the point on the curve whose abscissa is

(A) $\frac{p+q}{2}$ (B) $\frac{p-q}{2}$ (C) $\frac{pq}{2}$ (D) $\frac{p}{2}$

3. The least value of k for which the function $f(x)=x^2+kx+1$ is a increasing function in the interval $1\le x\le 2$

(A) –1 (B) –2 (C) 1 (D) 3

4. The interval in which $f(x)=x^3-3x^2-9x+20$ is strictly decreasing

(A) $(-1,3)$ (B) $(3,\infty)$ (C) $(-\infty,-1)$ (D) $(5,9)$

5. The critical points of $f(x)=(x-2)^{\frac{2}{3}}(2x+1)$ are

(A) $(3,4)$ (B) $(5,6)$ (C) $(1,2)$ (D) $(0,5)$

6 The number of stationary points of $f(x)=\sin x$ in $[0,2\pi]$ are

(A) 0 (B) 1 (C) 2 (D) 3

7. local minimum values of the function $f(x)=x+\frac{1}{x}, x>0$

(A) 2 (B) -2 (C) 4 (D) 5

8. If the function $f(x)=\frac{a}{x}+x^2$, has maximum at $x=-3$, then the value of 'a' is

(A) 54 (B) -54 (C) 10 (D) -10

9. The point at which $f(x)=(x-1)^4$ assumes local maximum or local minimum value are

(A) 0 (B) 1 (C) 4 (D) -1

10. The global maximum and global minimum of $f(x)=2x^3-9x^2+12x+6$ in [0, 2]

(A) $(11,6)$ (B) $(6,11)$ (C) $(-6,11)$ (D) $(-11,6)$

11. Local maximum and local minimum values of the function $(x-1)(x+2)^2$ are

(A) – 4, 0 (B) 0, – 4 (C) 4, 0 (D) None of these

12. The angle between the curves y = x^3 and y = $e^{3(x-1)}$ at (1, 1) is

(A) 0 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

13. Which of the following is not a decreasing function on the interval $\left(0, \frac{\pi}{2}\right)$

(A) cos x (B) cos 2x (C) cos 3x (D) cot x

14. For the curve $x = t^2 - 1, y = t^2 - t,$ the tangent perpendicular to x-axis then

(A) $t = 0$ (B) $t = \frac{1}{2}$ (C) $t = 1$ (D) $t = \frac{1}{\sqrt{3}}$

15. For the function $f(x) = e^x, a = 0, b = 1,$ the value of c in mean value theorem will be

(A) $\log x$ (B) $\log(e-1)$ (C) 0 (D) 1

16. From mean value theorem $f(b) - f(a) = (b-a)f'(x_1); a < x_1 < b$ if $f(x) = \frac{1}{x},$ then x_1 =

(A) $\sqrt{ab}$ (B) $\frac{a+b}{2}$ (C) $\frac{2ab}{a+b}$ (D) $\frac{b-a}{b+a}$

17. The minimum of $f(x) = \frac{1+x+x^2}{1-x+x^2}$ occurs at x =

(A) – 1 (B) 1 (C) 2 (D) – 2

18. The minimum value of $(px + qy)$ when $xy = n^2$ is equal to

(A) $2n\sqrt{pq}$ (B) $2pq\sqrt{n}$ (C) $2\sqrt{npq}$ (D) $2pqn$

19. The greatest value of $f(x) = (x+1)^{1/3} - (x-1)^{1/3}$ on $[0,1]$ is

(A) 1 (B) 2 (C) 3 (D) 1/3

20. The points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangent is parallel x-axis is

(A) (1, 0), (–1, –4) (B) (0, –1), (–2, 3)
(C) (2, 13), (–2, –3) (D) (1, 2), (1, –2)

21. The value of 'c' in Lagrange's mean value theorem for f(x) = $\log x$ on [1, e] is

(A) $\frac{e}{2}$ (B) e – 1 (C) e – 2 (D) 1 – e

22. The value of 'c' in Lagrange's mean value theorem for $f(x) = x(x-2)^2$ in [0, 2] is

(A) 0 (B) 2 (C) $\frac{2}{3}$ (D) $\frac{3}{2}$

23. The equation of the tangent to the curve $6y = 7 - x^3$ at (1, 1) is

(A) 2x + y = 3 (B) x + 2y = 3 (C) x + y = –1 (D) x + y + 2 = 0

24. Rolle's theorem cannot be applicable for

(A) $f(x) = \sqrt{4 - x^2}$ in [–2, 2] (B) f(x) = [x] in [–1, 1]
(C) $f(x) = x^2 + 3x - 4$ in [–4, 1] (D) $f(x) = \cos 2x$ in $[0, \pi]$

25. If A > 0, B > 0, and A + B = $\frac{\pi}{3}$, then the maximum value of $\tan A \tan B$ is

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{3}$ (C) 3 (D) $\sqrt{3}$

26. Acute angle between the curves $y^2 = x$ and $x^2 = y$ at (1, 1) is

(A) $\frac{\pi}{2}$ (B) $\tan^{-1}\left(\frac{4}{3}\right)$ (C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $\tan^{-1}\left(\frac{3}{4}\right)$

27. Equation of the tangent to the curve $y = 1 - e^{\frac{x}{2}}$ at the point where the curve cuts *y*-axis is

(A) $x + y = 0$ (B) $x + 2y = 0$ (C) $2x + y = 0$ (D) $2x - y = 0$

28. The absolute maximum of $y = x^3 - 3x + 2$ in $0 \le x \le 2$ is

(A) 4 (B) 6 (C) 2 (D) 0

29. The slant height of a cone is fixed at 7cm. The rate of increase in the volume of the cone corresponding to the rate of increase of 0.3cm/s in the height when *h* = 4 cm is

(A) $\frac{\pi}{10} cc/s$ (B) $\frac{3\pi}{10} cc/s$ (C) $\frac{\pi}{5} cc/s$ (D) $\frac{7\pi}{10} cc/s$

30. A point on the parabola $y^2 = 18x$ at which the ordinate increases at twice the rate of the abscissa is

(A) (9/8, 9/2) (B) (2, –4) (C) (–9/8, 9/2) (D) (2, 4)

31. The approximate value of $(1.0002)^{3000}$ is

(A) 1.2 (B) 1.4 (C) 1.6 (D) 1.8

32. If the percentage error in measuring the surface area of a sphere is α%, then the error in its volume is

(A) $\frac{3}{2}\alpha\%$ (B) $\frac{2}{3}\alpha\%$ (C) $3\alpha\%$ (D) none of these

33. A rectangular sheet of fixed perimeter with sides having their length in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed square is 100, the resulting box has maximum volume, the dimensions of the sides of the rectangular sheet are

(A) 24, 45 (B) 32, 65 (C) 24, 60 (D) 32, 60

34. $f(x) = \sin x - ax$ is decreasing in R if

(A) a > 1 (B) a < 1 (C) a > $\frac{1}{2}$ (D) a < $\frac{1}{2}$

35. The maximum value of $f(x) = 100 - |45 - x|$ is

(A) 100 (B) 145 (C) 55 (D) 45

36. The length of the sub-normal at (–1, 4) on $y = 4x^2$ is

(A) 4 (B) 16 (C) 32 (D) 8

37. The length of sub-normal at x = a on the parabola $y^2 = 16x$ is

(A) 8 (B) $\frac{2}{\sqrt{a}}$ (C) 2a (D) a

38. If $f(x) = x^{3/2}(3x - 10),\ x \geq 0,$ then $f(x)$ is increasing in

(A) $(-\infty, -1) \cup (1, \infty)$ (B) $[2, \infty)$

(C) $(-\infty, -1) \cup [1, \infty)$ (D) $(-\infty, 0] \cup (2, \infty)$

39. Let $f(x) = \log(\sin x + \cos x),\ x \in x\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$. Then *f* is strictly increasing in the interval

(A) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (B) $\left(0, \frac{3\pi}{8}\right)$ (C) $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$

40. The smallest value of $x^2 - 3x + 3$ in the interval [–3, 3/2]

(A) $\frac{3}{4}$ (B) 5 (C) –15 (D) –20

41. If $\int \frac{\cos^4 x}{\sin^2 x} dx = A\cot x + B\sin 2x + C\frac{x}{2} + D,$ then

(A) B = - 2 (B) B = - 3/2 (C) B = -1/4 (D) B = – 1

42. $\int \{1 + 2\tan x(\tan x + \sec x)\}^{\frac{1}{2}} dx =$

(A) $\log|\sec x(\sec x - \tan x)| + c$ (B) $\log|\sec x(\sec x + \tan x)| + c$

(C) $\log\left|\frac{\sec x}{\sec x | \tan x}\right| + c$ (D) $\log|\cos x(\sec x + \tan x)| + c$

43. $\int \frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}$

(A) $\frac{\left(x^2+1\right)^{3/2}}{x^3}\left[\frac{2}{3}-\log\left(\frac{x^2+1}{x^2}\right)\right]+c$ (B) $\frac{\left(x^2+1\right)^{3/2}}{3x^3}\left[\log\left(\frac{x^2+1}{x^2}\right)-\frac{2}{3}\right]+c$

(C) $\frac{2\left(x^2+1\right)^{3/2}}{3x^3}\left[\frac{1}{3}-\frac{1}{2}\left(\frac{x^2+1}{x^2}\right)\right]+c$ (D) $\frac{\left(x^2+1\right)^{3/2}}{3x^3}\left[\log\left(\frac{x^2+1}{x^2}\right)+\frac{2}{3}\right]+c$

44. $\int \frac{mx^{m+2n-1}-nx^{n-1}}{x^{2m+2n}+2x^{m+n}+1}dx$ is equal to

(A) $\frac{x^m}{x^{m+n}+1}+c$ (B) $\frac{x^n}{x^{m+n}+1}+c$ (C) $\frac{x^{m+n}-1}{x^{m+n}+1}+c$ (D) $-\frac{x^n}{x^{m+n}+1}+c$

45. $\int \frac{x^5}{\sqrt{1+x^3}}dx=$

(A) $\frac{2}{9}\left(x^3-2\right)\sqrt{1+x^3}+c$ (B) $\frac{2}{9}\left(x^3+2\right)\sqrt{1+x^3}+c$

(C) $\frac{1}{9}\left(x^3-1\right)\sqrt{1+x^3}+c$ (D) $\frac{1}{9}\left(x^3+1\right)\sqrt{1+x^3}+c$

46. $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx=$

(A) $\tan^{-1}\left(\tan^2 x\right)+c$ (B) $\tan^{-1}\left(\cos^2 x\right)+c$ (C) $\tan^{-1}\left(\sin^2 x\right)+c$ (D) $\tan^{-1}\left(\cot^2 x\right)+c$

47. $\int \frac{x}{x-\sqrt{x^2-1}}dx$ is equal to

(A) $\frac{x^3}{3}+\frac{1}{2}\left(x^2-x-1\right)^{\frac{3}{2}}+c$ (B) $\frac{x^3}{3}+\frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}}+c$

(C) $\frac{x^3}{3}+\frac{2}{3}\left(x^2-1\right)^{\frac{3}{2}}+c$ (D) $\frac{x^3}{3}+\left(x^2-1\right)^{\frac{3}{2}}+c$

48. $\int \frac{\left(x^2-1\right)}{\left(x^4+3x^2+1\right)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx=$

(A) $\log\left|\tan^{-1}\left(\frac{x-1}{x}\right)\right|+c$ (B) $\log\left|\tan^{-1}\left(x+\frac{1}{x}\right)\right|+c$

(C) $\log\left|\tan^{-1}\left(x^2+1\right)\right|+c$ (D) $\log\left|\tan^{-1}\left(\frac{x+1}{x}\right)\right|+c$

49. $\int \frac{\cos x}{\cos 3x}dx=$

(A) $\frac{1}{2\sqrt{3}}\log\left|\frac{1+\tan x}{1-\tan x}\right|+c$ (B) $\frac{1}{2\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right|+c$

(C) $\frac{1}{\sqrt{3}}\log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right|+c$ (D) $\frac{2}{\sqrt{3}}\log\left|\frac{1-\tan x}{1+\tan x}\right|+c$

50. If $l^r(x)$ means log log log ….. logx, the log being repeated ‘r’ times, then

$\int\left\{xl(x)l^2(x)l^3(x)....l^r(x)\right\}^{-1}dx=$

(A) $l^{r+1}(x)+c$ (B) $\frac{l^{r+1}(x)+c}{r+1}+c$ (C) $l^r(x)+c$ (D) $\frac{l^r(x)+c}{r}$

51. $\int\sqrt{\dfrac{\cos x-\cos^3 x}{1-\cos^3 x}}dx=$

(A) $\dfrac{2}{3}\sin^{-1}\left(\cos^{\frac{3}{2}} x\right)+c$ (B) $\dfrac{3}{2}\sin^{-1}\left(\cos^{\frac{3}{2}} x\right)+c$

(C) $\dfrac{2}{3}\cos^{-1}\left(\cos^{\frac{3}{2}} x\right)+c$ (D) $\dfrac{3}{2}\cos^{-1}\left(\cos^{\frac{3}{2}} x\right)+c$

52. $\int\dfrac{\sin^6 x+\cos^6 x}{\sin^2 x\cos^2 x}dx=$

(A) $\tan x+\cot x+c$ (B) $\tan x+\cot x+3x+c$

(C) $\tan x-\cot x-3x+c$ (D) $\tan x+\cot x-3x+c$

53. $\int\dfrac{2\cos x-3\sin x}{6\cos x+4\sin x}dx$

(A) $\dfrac{1}{2}\ln\left(6\cos x-4\sin x\right)+c$ (B) $\dfrac{1}{2}\ln\left(4\cos x+6\sin x\right)+c$

(C) $\dfrac{1}{2}\ln\left(4\cos x-6\sin x\right)+c$ (D) $\dfrac{1}{2}\ln\left(6\cos x+4\sin x\right)+c$

54. $\int\dfrac{dx}{\cos^4 x\sin^2 x}=$

(A) $\dfrac{1}{2}\tan^3 x+\tan x-\cot x+c$ (B) $\dfrac{1}{3}\tan^3 x+2\tan x+\cot x+c$

(C) $\dfrac{1}{3}\tan^3 x+2\tan x-\cot x+c$ (D) $\dfrac{1}{3}\tan^3 x-2\tan x-\cot x+c$

55. $\int\dfrac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx=$

(A) $\dfrac{1}{2}\cos 2x+c$ (B) $\dfrac{-1}{2}\cos 2x+c$ (C) $\dfrac{-1}{\left(1+\tan x\right)^2}+c$ (D) $\dfrac{-1}{2}\sin 2x+c$

56. $\int\dfrac{\cos 2x-\sin 2x}{\cos 2x+\sin 2x}dx=$

(A) $-\dfrac{1}{2}\log\left|\cos\left(\dfrac{\pi}{4}-2x\right)\right|+c$ (B) $\dfrac{1}{2}\log\left|\cos\left(\dfrac{\pi}{4}-2x\right)\right|+c$

(C) $\tan 2x+c$ (D) $\cot 2x+c$

57. $\int\dfrac{1}{x\sqrt{1+x^n}}dx=$

(A) $\dfrac{1}{n}\log\sqrt{\dfrac{1+x^n}{1-x^n}}+c$ (B) $\dfrac{1}{n}\log\left|\dfrac{\sqrt{1+x^n}-1}{\sqrt{1-x^n}+1}\right|+c$ (C) $\dfrac{1}{n}\log\left|\dfrac{1+x^n}{1-x^n}\right|+c$ (D) $\dfrac{1}{n}\log\left|\dfrac{\sqrt{1+x^n}-1}{\sqrt{1+x^n}+1}\right|+c$

58. $\int\sin^5 \mathrm{x}.\cos^{100}\mathrm{x}\,\mathrm{dx}=$

(A) $\dfrac{\cos^{105}x}{105}+2\dfrac{\cos^{103}x}{103}+\dfrac{\cos^{101}x}{101}+c$ (B) $-\dfrac{\cos^{105}x}{105}+2\dfrac{\cos^{103}x}{103}-\dfrac{\cos^{101}x}{101}+c$

(C) $-\dfrac{\cos^{105}x}{105}-2\dfrac{\cos^{103}x}{103}+\dfrac{\cos^{101}x}{101}+c$ (D) $\dfrac{\cos^{105}x}{105}-2\dfrac{\cos^{103}x}{103}+\dfrac{\cos^{101}x}{101}+c$

59. $\int\sqrt{\cos x}.\sin^3 x.dx$

(A) $-\dfrac{2}{3}\cos^{3/2}x+\dfrac{2}{7}\cos^{7/2}x+c$ (B) $\dfrac{2}{3}\cos^{3/2}x+\dfrac{2}{7}\cos^{7/2}x+c$

(C) $\dfrac{1}{3}\cos^{3/2}x-\dfrac{2}{7}\cos^{7/2}x+c$ (D) $\dfrac{1}{3}\cos^{3/2}x+\dfrac{2}{7}\cos^{7/2}x+c$

60. $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx =$

(A) $\log|10^x + x^{10}| + c$

(B) $-\log|10^x + x^{10}| + c$

(C) $10^x + x^{10} + c$

(D) $\log|10^x + x^9| + c$

61. $\int \sec x . \log(\sec x + \tan x) dx =$

(A) $[\log(\sec x + \tan x)]^2 + c$

(B) $\frac{[\log(\sec x + \tan x)]^2}{2} + c$

(C) $-\log(\sec x + \tan x) + c$

(D) $\log(\sec x + \tan x) + c$

62. $\int \frac{1 - \tan x}{1 + \tan x} dx =$

(A) $\log|1 + \tan x| + c$

(B) $\log|1 - \tan x| + c$

(C) $\log|\sin x + \cos x| + c$

(D) $\log|\sin x - \cos x| + c$

63. $\int \frac{x^2}{\sqrt{1 - x^6}} dx =$

(A) $\frac{1}{3}\sin^{-1}(x^3) + c$

(B) $\frac{1}{3}\cos^{-1}(x^3) + c$

(C) $-\frac{1}{3}\sin^{-1}(x^3) + c$

(D) $\frac{1}{4}\cos^{-1}(x^3) + c$

64. $\int x^2 e^x dx =$

(A) $e^x(x^2 - 2x + 2) + c$

(B) $e^x(x^2 + 2x + 2) + c$

(C) $x^2 + ex + c$

(D) $e^2(x^2 + x + 2) + c$

65. $\int \frac{x + \sin x}{1 + \cos x} dx =$

(A) $x \tan \frac{x}{2} + c$

(B) $x \cot \frac{x}{2} + c$

(C) $x \sin \frac{x}{2} + c$

(D) $x \cos \frac{x}{2} + c$

66. $\int e^x \left[\frac{x + 4}{(x + 6)^3}\right] dx =$

(A) $\frac{e^x}{(x + 6)^2} + c$

(B) $e^x \frac{1}{(x + 4)^2} + c$

(C) $e^x \frac{x}{x + 6} + c$

(D) $e^x \frac{4}{(x + 6)^2} + c$

67. $\int e^{\sin^{-1} x}\left[1 + \frac{x}{\sqrt{1 - x^2}}\right] dx =$

(A) $x e^{\sin^{-1} x} + c$

(B) $e^{\sin^{-1} x} + c$

(C) $\frac{1}{\sqrt{1 - x^2}} e^{\sin^{-1} x} + c$

(D) $x^2 e^{\sin^{-1} x} + c$

68. $\int \frac{ax + b}{cx + d} dx =$

(A) $\frac{ax}{c} - \frac{bc - ad}{c^2}\log|cx + d| + c$

(B) $\frac{ax}{c} + \frac{bc - ad}{c^2}\log|cx + d| + c$

(C) $\log\left|\frac{ax + b}{cx + d}\right| + c$

(D) $\frac{ax}{c} + \frac{b}{c}\log|cx + d| + c$

69. $\int \sec^3 x \, dx =$

(A) $\frac{1}{3}\sec^3 x - x + c$

(B) $\frac{1}{2}\sec^2 x \tan x + x + c$

(C) $\frac{1}{2}\tan x \sec x + \frac{1}{2}\log|\sec x + \tan x| + c$

(D) $-\frac{1}{2}\tan x \sec x + \frac{1}{2}\log|\sec x + \tan x| + c$

70. $\int 5^{\log_e x} dx$ is equal to

(A) $\frac{x^{(\log_e 5+1)}}{\log_e 5+1}+k$ (B) $5^{\log_e x}.\log 5$

(C) $e^{\log_5 x}+k$ (D) $\frac{5^{\log_e x}}{x}+k$

71. $\int \sin x \log(\sec x+\tan x)dx = f(x)+x+c$ then $f(x)=$

(A) $\cos x\log(\sec x+\tan x)+c$ (B) $\sin x\log(\sec x+\tan x)+c$

(C) $-\cos x\log(\sec x+\tan x)+c$ (D) $-\cos x\log\sec x+c$

72. $\int \frac{x^2(x\sec^2 x+\tan x)}{(x\tan x+1)^2}dx = \frac{-x^2}{x\tan x+1}+f(x)+c$ then $f(x=)$

(A) $\cos x\log(\sec x+\tan x)+c$ (B) $\sin x\log(\sec x+\tan x)+c$

(C) $2\log|x\sin x+\cos x|+c$ (D) $2\log|x\cos x+\sin x|+c$

73. $\int x^3(\log x)^2 dx =$

(A) $\frac{x^4}{4}\left[(\log x)^2-\frac{\log x}{2}+\frac{1}{8}\right]+c$ (B) $\frac{x^4}{4}\left[(\log x)^2+\frac{\log x}{2}+\frac{1}{8}\right]+c$

(C) $(\log x)^2+2\log x+8+c$ (D) $\frac{x^4}{4}\left[(\log x)^2-\frac{\log x}{2}-\frac{1}{8}\right]+c$

74. $\int \frac{dx}{x(x^n+1)}$ is equal to

(A) $\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right)+c$ (B) $\frac{1}{n}\log\left(\frac{x^n+1}{x^n}\right)+c$ (C) $\log\left(\frac{x^n}{x^n+1}\right)+c$ (D) $\log\left(\frac{x^n+1}{x^n}\right)+c$

75. $\int\left(\frac{x+2}{x+4}\right)^2 e^x dx$ is equal to

(A) $e^x\left(\frac{x}{x+4}\right)+c$ (B) $e^x\left(\frac{x+2}{x+4}\right)+c$ (C) $e^x\left(\frac{x-2}{x+4}\right)+c$ (D) $e^x\left(\frac{2xe^2}{x+4}\right)+c$

76. If $\int \frac{\cos 4x+1}{\cot x-\tan x}dx = A\cos 4x+B$, then

(A) A = –1/2 (B) A = –1/8 (C) A = –1/4 (D) A= –1/6

77. $\int \frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx$, $\alpha\neq n\pi$, $n\in Z$ is equal to

(A) $-2\operatorname{cosec}\alpha(\cos\alpha-\tan x\sin\alpha)^{1/2}+C$ (B) $-2(\cos\alpha+\cot x\sin\alpha)^{1/2}+C$

(C) $-2\operatorname{cosec}\alpha(\cos\alpha+\cot x\sin\alpha)^{1/2}+C$ (D) $-2\operatorname{cosec}\alpha(\sin\alpha+\cot x\cos\alpha)^{1/2}+C$

78. $\int e^x\left[\frac{x+4}{(x+6)^3}\right]dx =$

(A) $\frac{e^x}{(x+6)^2}+c$ (B) $e^x\frac{1}{(x+4)^2}+c$ (C) $e^x\frac{x}{x+6}+c$ (D) $e^x\frac{4}{(x+6)^2}+c$

79. $\int e^{\sin^{-1}x}\left[1+\frac{x}{\sqrt{1-x^2}}\right]dx =$

(A) $xe^{\sin^{-1}x}+c$ (B) $e^{\sin^{-1}x}+c$ (C) $\frac{1}{\sqrt{1-x^2}}e^{\sin^{-1}x}+c$ (D) $x^2e^{\sin^{-1}x}+c$

80. If $\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x} + c$, then

(A) a = –1, b = 1/3 (B) a = –3, b = 2/3 (C) a = –2, b = 4/3 (D) a = –2, b = 2/3

81. $\int \frac{\ln(\tan x)}{\sin x \cos x} dx$ is equal to

(A) $\frac{1}{2}\ln(\tan x) + c$ (B) $\frac{1}{2}\ln(\tan^2 x) + c$ (C) $\frac{1}{2}(\ln(\tan x))^2 + c$ (D) None of these

82. $\int \frac{\sqrt{x-1}}{x\sqrt{x+1}} dx$ is equal to

(A) $\ln\left|x - \sqrt{x^2-1}\right| - \tan^{-1} x + c$ (B) $\ln\left|x + \sqrt{x^2-1}\right| - \tan^{-1} x + c$

(C) $\ln\left|x - \sqrt{x^2-1}\right| - \sec^{-1} x + c$ (D) $\ln\left|x + \sqrt{x^2-1}\right| - \sec^{-1} x + c$

83. $\int \sqrt{e^x - 1}\, dx$ is equal to

(A) $2\left[\sqrt{e^x-1} - \tan^{-1}\sqrt{e^x-1}\right] + c$ (B) $\sqrt{e^x-1} - \tan^{-1}\sqrt{e^x-1} + c$

(C) $\sqrt{e^x-1} + \tan^{-1}\sqrt{e^x-1} + c$ (D) $2\left[\sqrt{e^x-1} + \tan^{-1}\sqrt{e^x-1}\right] + c$

84. $\int \frac{e^{3x} + e^x}{e^{4x} - e^{2x} + 1} dx =$

(A) $\frac{1}{4}\log\left(e^{4x} - e^{2x} + 1\right) + c$ (B) $\tan^{-1}\left(e^x - e^{-x}\right) + c$

(C) $\tan^{-1}\left(e^x + e^x\right) + c$ (D) $\tan^{-1}\left(e^{-x} - e^x\right) + c$

85. If $\int \frac{dx}{x^4 + 5x^2 + 4} = A\tan^{-1} x + B\tan^{-1}\left(\frac{x}{2}\right) + C$, then B =

(A) 1/6 (B) –1/6 (C) 6 (D) –6

86. $\int \frac{x^2+2}{(x^2+1)(x^2+4)} dx = A\tan^{-1} x + B\tan^{-1}\frac{x}{2} + c$ then (A, B)

(A) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (B) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (C) $\left(\frac{-1}{2}, \frac{-2}{3}\right)$ (D) $\left(\frac{1}{3}, \frac{1}{3}\right)$

87. If $\int \frac{6x+7}{(x+2)^2} dx = A\log|x+2| + \frac{B}{x+2} + c$ then $(A, B) =$

(A) $\left(6, \frac{1}{5}\right)$ (B) $(-6, -5)$ (C) $\left(\frac{1}{6}, -5\right)$ (D) $(6, 5)$

88. $\int \frac{x^3-1}{x^3+x} dx$ equal to

(A) $x - \log x + \log(x^2+1) - \tan^{-1} x + c$ (B) $x - \log x + \frac{1}{2}\log(x^2+1) - \tan^{-1} x + c$

(C) $x + \log x + \frac{1}{2}\log(x^2+1) + \tan^{-1} x + c$ (D) $x + \log x - \frac{1}{2}\log(x^2+1) - \tan^{-1} x + c$

89. $\int \frac{dx}{1 + \sqrt[3]{x+1}} =$

(A) $\frac{3}{2}(x+1)^{2/3} - 3(x+1)^{1/3} + 3\log\left|1 + \sqrt[3]{x+1}\right| + c$ (B) $\frac{3}{2}(x+1)^{2/3} + 3(x+1)^{1/3} + 3\log\left|1 + \sqrt[3]{x+1}\right| + c$

(C) $\frac{3}{2}(x+1)^{2/3} + 3(x+1)^{1/3} + c$ (D) $(x+1)^{2/3} + 3(x+1)^{1/3} + c$

90. $\int \frac{ax+b}{cx+d}dx =$

(A) $\frac{ax}{c}-\frac{bc-ad}{c^2}\log|cx+d|+c$

(B $\frac{ax}{c}+\frac{bc-ad}{c^2}\log|cx+d|+c$

(C) $\log\left|\frac{ax+b}{cx+d}\right|+c$

(D) $\frac{ax}{c}+\frac{b}{c}\log|cx+d|+c$

91. If $\int(\sin 2x-\cos 2x)dx=\frac{1}{\sqrt{2}}\sin(2x-a)+c$ then

(A) $a=\frac{7\pi}{4}$ (B) $a=-\frac{5\pi}{4}$ (C) $a=-\frac{\pi}{4}$ (D) $a=\frac{-3\pi}{4}$

92. $\int \frac{1}{x^2}\sqrt{\frac{x-1}{x+1}}dx =$

(A) $\cos^{-1}\frac{1}{|x|}-\frac{\sqrt{x^2-1}}{x}+x$

(B) $\cos^{-1}\frac{1}{|x|}+\frac{\sqrt{x^2-1}}{x}+x$

(C) $\sin^{-1}\frac{1}{|x|}-\frac{\sqrt{x^2-1}}{x}+x$

(D) $\sin^{-1}\frac{1}{|x|}+\frac{\sqrt{x^2-1}}{x}+x$

93. $\int e^{3\log x}(x^4+1)^{-1}dx$

(A) $\frac{1}{4}\ln(x^2+1)+c$ (B) $\frac{1}{8}\ln(x^3+1)+c$ (C) $\frac{1}{4}\ln(x^6+1)+c$ (D) $\frac{1}{4}\ln(x^4+1)+c$

94. $\int \sec^3 x\,dx =$

(A) $\frac{1}{3}\sec^3 x - x + c$

(B) $\frac{1}{2}\sec^2 x\tan x + x + c$

(C) $\frac{1}{2}\tan x\sec x+\frac{1}{2}\log|\sec x+\tan x|+c$

(D) $-\frac{1}{2}\tan x\sec x+\frac{1}{2}\log|\sec x+\tan x|+c$

95. $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ equals

(A) $\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$

(B) $\frac{1}{2}\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+c$

(C) $\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$

(D) $\log\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+c$

96. The value of $\sqrt{2}\int \frac{\sin x\,dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is

(A) $x+\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

(B) $x-\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(C) $x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

(D) $x-\log\left|\cos\left(x-\frac{\pi}{4}\right)\right|+c$

97. If $\int f(x)dx=\Psi(x)$, then $\int x^5 f(x^3)dx$ is equal to

(A) $\frac{1}{3}x^3\Psi(x^3)-\int x^2\Psi(x^3dx)+C$

(B) $\frac{1}{3}[x^3\Psi(x^3)-\int x^3\Psi(x^3)dx]+C$

(C) $\frac{1}{3}[x^3\Psi(x^3)-2\int x^3\Psi(x^3)dx]+C$

(D) $\frac{1}{3}x^3\Psi(x^3)-3\int x^3\Psi(x^3)dx+C$

98. The integration $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

(A) $(x-1)e^{x+\frac{1}{x}}+c$ (B) $xe^{x+\frac{1}{x}}+c$ (C) $(x+1)e^{x+\frac{1}{x}}+c$ (D) $-xe^{x+\frac{1}{x}}+c$

99. $\int e^x\frac{x^2+1}{(x+1)^2}dx=$

(A) $e^x\left(\frac{x-1}{x+1}\right)+c$ (B) $e^x\left(\frac{x+1}{x-1}\right)+c$ (C) $e^x\frac{x-1}{(x+1)^2}+c$ (D) $e^x\frac{x+1}{(x-1)^2}+c$

100. $\int e^x\left[\frac{x^3+x+1}{(1+x^2)^{3/2}}\right]dx=$

(A) $e^x\frac{x}{\sqrt{1+x^2}}+c$ (B) $e^x.\frac{e^x}{\sqrt{1+x^2}}+c$ (C) $e^x+\frac{x}{\sqrt{1+x^2}}+c$ (D) $e^x\sqrt{1+x^2}+c$

101. Value of the integral $\mathrm{I}=\int_0^1 x(1-x)^n\,dx=$

(A) $\frac{1}{n+2}$ (B) $\frac{1}{n+1}-\frac{1}{n+2}$ (C) $\frac{1}{n+1}+\frac{1}{n+2}$ (D) $\frac{1}{n+1}$

102. If $f(a+b-x)=f(x)$, then $\int_a^b x.f(x)\,dx=$

(A) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (B) $\frac{a+b}{2}f(x)dx$

(C) $\frac{b-a}{2}\int_a^b f(x)dx$ (D) $(a+b)\int_a^b f(x)dx$

103. $\int_a^b\frac{f(x)}{f(x)+f(a+b-x)}dx=$

(A) $a-b$ (B) $b-a$ (C) $\frac{b-a}{2}$ (D) $\frac{a-b}{2}$

104. $\int_0^n[x]dx=$

(A) n (B) $\frac{n(n+1)}{2}$ (C) $\frac{n(n-1)}{2}$ (D) 0

105. $\int_0^a\sqrt{\frac{a+x}{a-x}}\,dx=$

(A) $\frac{a}{2}(\pi+2)$ (B) $\frac{a}{2}(\pi-2)$ (C) $\frac{a}{3}(\pi+2)$ (D) $\frac{a}{2}(\pi+3)$

106. $\int_0^1\frac{dx}{e^x+e^{-x}}=$

(A) $\tan^{-1}e$ (B) $\frac{\pi}{4}$ (C) $\tan^{-1}e-\frac{\pi}{4}$ (D) $\tan^{-1}e+\frac{\pi}{4}$

107. $\int_1^2\log x\,dx=$

(A) $2\log 2-1$ (B) $\log 2-1$ (C) $2\log 2+1$ (D) $2\log 2-2$

108. $\int_0^1 \frac{xe^x}{(x+1)^2}dx =$

(A) $\frac{e}{2}$ (B) $\frac{e-1}{2}$ (C) $\frac{e}{2}-1$ (D) $\frac{e-3}{2}$

109. $\int_0^1 \sqrt{x(1-x)}dx =$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$

110. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx =$

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

111. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(A) $\frac{3}{2}$ (B) 2 (C) 1 (D) $\frac{1}{2}$

112. $\int_{-a}^{a} x|x|dx =$

(A) $\frac{a}{3}$ (B) $\frac{a^2}{3}$ (C) $\frac{a^2}{2}$ (D) 0

113. The function $f(x)=\int_0^x \log\left(t+\sqrt{1+t^2}\right)dt$ is

(A) a periodic function (B) an even function
(C) an odd function (D) even or odd

114. $\int_{\pi}^{10\pi}|\sin x|dx =$

(A) 20 (B) 8 (C) 10 (D) 18

115. $\int_{-\pi/2}^{\pi/2} \sin^4 x.\cos^6 x\,dx =$

(A) $\frac{3\pi}{128}$ (B) $\frac{3\pi}{256}$ (C) $\frac{5\pi}{128}$ (D) $\frac{7\pi}{128}$

116. $\int \sec x.\log(\sec x+\tan x)dx =$

(A) $[\log(\sec x+\tan x)]^2+c$ (B) $\frac{[\log(\sec x+\tan x)]^2}{2}+c$

(C) $-\log(\sec x+\tan x)+c$ (D) $\log(\sec x+\tan x)+c$

117. $\int \frac{x^2}{\sqrt{1-x^6}}dx =$

(A) $\frac{1}{3}\sin^{-1}(x^3)+c$ (B) $\frac{1}{3}\cos^{-1}(x^3)+c$

(C) $-\frac{1}{3}\sin^{-1}(x^3)+c$ (D) $\frac{1}{4}\cos^{-1}(x^3)+c$

118. $\int x^2 e^x \, dx =$

(A) $e^x(x^2 - 2x + 2) + c$ (B) $e^x(x^2 + 2x + 2) + c$

(C) $x^2 + ex + c$ (D) $e^2(x^2 + x + 2) + c$

119. $\int \log_{10} x \, dx$ is equal to

(A) $(x-1)\log_e x + c$ (B) $\log_e 10.x \log_e\left(\frac{x}{e}\right) + c$

(C) $\log_{10} e.x \log_e\left(\frac{x}{e}\right) + c$ (D) $\frac{1}{x} + c$

120. $\int x \tan x \sec^2 x \, dx =$

(A) $\frac{1}{2}[x\tan^2 x - \tan x + x] + c$ (B) $\frac{1}{2}[x\tan^2 x - \tan x - x] + c$

(C) $\frac{1}{2}[x\tan^2 x + \tan x - x] + c$ (D) $x\tan^2 x - \tan x + x + c$

121. If $\int e^x\left(f(x) - f'(x)\right)dx = \phi(x)$, then $\int e^x f(x)\,dx =$

(A) $\phi(x) + e^x f(x)$ (B) $\phi(x) - e^x f(x)$

(C) $\frac{1}{2}\left\{\phi(x) + e^x f(x)\right\}$ (D) $\frac{1}{2}\left\{\phi(x) - e^x f'(x)\right\}$

122. $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx =$

(A) $e^x \tan\frac{x}{2} + c$ (B) $-e^x \cot\frac{x}{2} + c$

(C) $e^x \cos\frac{x}{2} + c$ (D) $e^{-x} \cot\frac{x}{2} + c$

123. If $\int \frac{dx}{(x+2)\left(x^2+1\right)} = a\ln\left(1+x^2\right) + b\tan^{-1}x + \frac{1}{5}\ln|x+2| + C$, then

(A) $a = -\frac{1}{10}, b = -\frac{2}{5}$ (B) $a = \frac{1}{10}, b = -\frac{2}{5}$ (C) $a = -\frac{1}{10}, b = \frac{2}{5}$ (D) $a = \frac{1}{10}, b = \frac{2}{5}$

124. $\int \frac{x + \sin x}{1 + \cos x} dx =$

(A) $x\tan\frac{x}{2} + c$ (B) $x\cot\frac{x}{2} + c$ (C) $x\sin\frac{x}{2} + c$ (D) $x\cos\frac{x}{2} + c$

125. $\int x^2 2^{3x} \, dx =$

(A) $\frac{2^{3x}x^2}{3\log 2} - \frac{x.2^{3x}}{9.(\log 2)^2} + \frac{2^{3x}}{27(\log 2)^3} + c$ (B) $\frac{x^2 2^{3x}}{3\log 2} - \frac{x.2^{3x+1}}{9.(\log 2)} + \frac{2^{3x+1}}{27(\log 2)} + c$

(C) $\frac{x^2 2^{3x}}{3\log 2} - \frac{2^{3x+1}.x}{3^2(\log 2)^2} + \frac{2^{3x+1}}{3^3(\log 2)^3} + c$ (D) $\frac{x^2 2^{3x}}{\log 2} - \frac{2^{3x+1}.x}{3^2(\log 2)^2} + \frac{2^{3x+1}}{(\log 2)^3} + c$.

PHYSICS

Syllabus: *MAGENETISM AND OPTICS:– 1. MAGNETISM AND MATTER, 2. RAY OPTICS, 3. WAVE OPTICS.*

1. Light falls on a prism of angle $\cos^{-1}\left(\frac{1}{8}\right)$ .If the minimum value of refractive index of the prism – material, so that light entering one face may not exit through the other is $\frac{k}{\sqrt{7}}$. Then k = ?

(A) 1 (B) 2 (C) 3 (D) 4

2. A fish under water at a depth of 20 cm can see the outer atmosphere through an aperture of radius of (critical angle of water is 45^0)

(A) 5 cm (B) 20 cm (C) 10 cm (D) $20/\sqrt{3}$

3. The refractive indices of glass and water are 3/2 and 4/3 respectively. The refractive index of glass with respect to water is

(A) 2 (B) 8/9 (C) 9/8 (D) 5/3

4. A fish under water at a depth of 20 cm can see the outer atmosphere through an aperture of radius of (critical angle of water is 45^0)

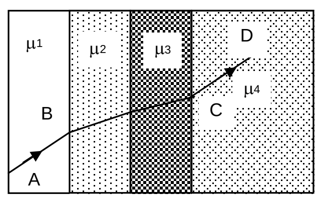
(A) 5 cm (B) 20 cm (C) 10 cm (D) $20/\sqrt{3}$

5. The angle of deviation when light is incident at an angle of 45^0 on one of the refracting faces of an equilateral prism of refractive index 1.414 is

(A) 40^0 (B) 30^0 (C) 45^0 (D) 50^0

6. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have

(A) $\mu_1 = \mu_2$ (B) $\mu_2 = \mu_3$ (C) $\mu_3 = \mu_4$ (D) $\mu_4 = \mu_1$



7. A fish looking up through the water sees the outside would contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the surface. The radius of this circle in cm is:

(A) $\frac{36}{\sqrt{7}}$ (B) $36\sqrt{7}$ (C) $4\sqrt{5}$ (D) $36\sqrt{5}$

8. The angle of deviation when light is incident at an angle of 45^0 on one of the refracting faces of an equilateral prism of refractive index 1.414 is

(A) 40^0 (B) 30^0 (C) 45^0 (D) 50^0

9. In case of refraction of light

(a) frequency changes (b) phase changes
(c) speed changes (d) wavelength changes
(A) a is correct (B) c and d are correct
(C) d, b, c are correct (D) a and b are correct

10. The critical angle of light going from medium A to medium B is θ. The speed of light in medium A is v. The speed of light in medium B is

(A) $\frac{v}{\sin\theta}$ (B) $\frac{v}{\cos\theta}$ (C) $v\sin\theta$ (D) $v\cos\theta$

11. When the beam of light travels through a medium which has lesser velocity of light then the value of wavelength and frequency will respectively

(A) increase, decrease (B) increase, unchanged
(C) decrease, unchanged (D) decrease, decrease

12. A layer of benzene (μ =1.50) 6 cm deep floats on water (μ=4/3) 4 cm deep. When viewed vertically through air, the apparent distance of the bottom of the vessel below the free surface of the benzene is

(A) 14 cm (B) 5 cm (C) 0 7 cm (D) 3.5 cm

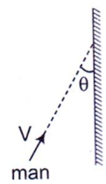
13. A ray of light passes from vacuum into a medium of refractive index μ, then angle of incidence is found to be twice the angle of refraction. The angle of incidence is:

(A) $\cos^{-1}(\mu/2)$ (B) $2\cos^{-1}(\mu/2)$ (C) $2\sin^{-1}(\mu)$ (D) $2\sin^{-1}(\mu/2)$

14. A plane mirror is made of glass slab $(\mu_g = 1.5)$ 2.5 cm thick and silvered on back. A point object is placed 5 cm in front of the unsilvered face of the mirror. What will be the position of final image?

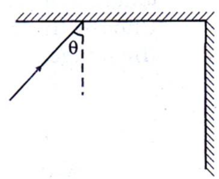
(A) 12 cm from unsilvered face (B) 14.6 cm from unsilvered face
(C) 5.67 cm from unsilvered face (D) 8.33 cm from unsilvered face

15. A person walks at a velocity v in a straight line forming an angle θ with the plane of a plane mirror. The velocity v_{rel} with which he approaches his image?



(A) $2v\sin\theta$ (B) $v\sin\frac{\theta}{2}$ (C) $2v\cos\theta$ (D) $v\cos\frac{\theta}{2}$

16. Two plane mirrors are arranged at right angles to each other as shown in figure. A ray of light is incident on the horizontal mirror at an angle θ. For what value of θ the ray emerges parallel to the incoming ray after reflection from the vertical mirror?

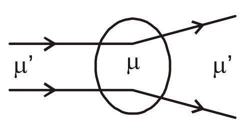


(A) 60° (B) 30° (C) 45° (D) All of these

17. Critical angle of glass is θ_1 and that of water is θ_2. The critical angle for water and glass surface would be $(\mu_g = 3/2, \mu_w = 4/3)$

(A) less than θ_2 (B) between θ_1 and θ_2
(C) greater than θ_2 (D) less than θ_1

18. In the diagram shown



(A) $\mu' = \mu$ (A) $\mu' < \mu$ (C) $\mu' > \mu$ (D) $\mu' \geq \mu$

19. The equiconvex lens, shown in the figure, has a focal length f. What will be the focal length of each half if the lens is cut along AB?

(A) $\frac{f}{2}$ (B) f (C) $\frac{3f}{2}$ (D) 2f

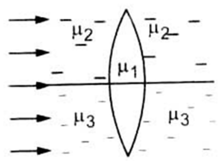
20. Two lenses of power - 15 D and + 5 D are in contact with each other. The focal length of the combination is:
(A) - 20 cm (B) - 10 cm (C) + 20 cm (D) + 10 cm

21. Focal length of a lens in air is f. Refractive index of the lens is μ. Focal length charges to f_1 if lens is immersed in a liquid of refractive index $\mu/2$ and it becomes f_2 if the lens is immersed in a liquid of refractive index 2μ. Then
(A) $f_1 = \frac{f}{2}$ (B) $f_2 = -2f$ (C) $f_2 = -\frac{3f}{2}$ (D) data is insufficient

22. A glass converging lens with a refractive index of 1.5 has a focal length of f in air. When it is completely immersed in a liquid of refractive index 2, its focal length and nature will be
(A) 2f, converging (B) 3f, converging (C) 2f, diverging (D) none of these

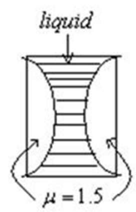
23. Two thin lenses of focal length 20 cm and 25 cm are placed in contact. The effective power of the combination is:
(A) 9D (B) 2D (C) 3D (D) none of these

24. A double convex lens, made of a material of refractive index μ_1, is placed inside two liquids of refractive indices μ_2 and μ_3, as shown. $\mu_2 > \mu_1 > \mu_3$. A wide, parallel beam of light is incident on the lens from the left. The lens will give rise to



(A) a single convergent beam (B) two different convergent beams
(C) two different divergent beams (D) a convergent and a divergent beam

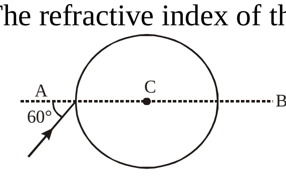
25. The effective focal length of the lens combination shown in the figure is – 60 cm. The radii of curvature of the curved surfaces of the plano-convex lenses are 12cm each and refractive index of the material of the lens is1.5.therefractiveindexoftheliquidis



(A) 1.33 (B) 1.42 (C) 1.53 (D) 1.60

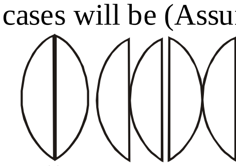
26. A plano-convex lens of thickness 3cm and radius of curvature 5cm when seen normally through the flat surface the thickness is found to be2cm.Then RI of material is
(A) 3/2 (B) 2/3 (C) 6/3 (D) none of these

27. A ray of light falls on a transparent sphere with center at C as shown in figure. The ray emerges from the sphere parallel to line AB. The refractive index of the sphere is



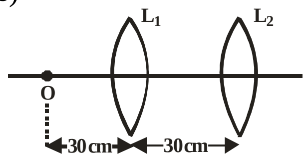
(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 3/2 (D) none of these

28. Two similar plano-convex lenses are combined together in three different ways as shown in figure. The ratio of the focal lengths in three cases will be (Assume this lens)



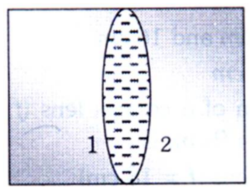
(A) 2 : 2 : 1 (B) 1 : 1 : 1 (C) 1 : 2 : 2 (D) 2 : 1 : 1

29. Two thin convex lenses are separated as shown. The focal length of lens L_1 is 15 cm and that of lens L_2 is 30 cm. An object is placed at distance 30 cm from lens L_1. The location of image formed finally will be at: (Assume thin lenses)



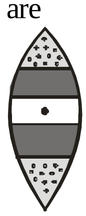
(A) infinity
(B) 15 cm behind the lens L_1
(C) optical centre of lens L_2
(D) 30 cm away from lens L_2

30. Two plano-concave lenses (1 and 2) of glass of refractive index 1.5 have radii of curvature 20 cm and 20 cm. They are placed in contact with their curved surface towards each other and the space between them is filled with liquid of refractive index 4/3. Then the combination lens is (Assume thin lens)



(A) Convex of focal length 70 cm
(B) Concave of focal length 70 cm
(C) Concave of focal length 60 cm
(D) none of these

31. A convex lens is made up of three different materials as shown in the figure. For a point object placed on its axis, the number of images formed are

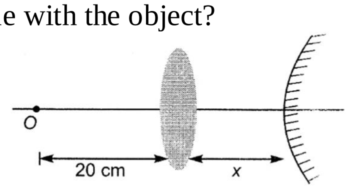


(A) 1 (B) 3 (C) 4 (D) 5

32. A denser medium of refractive index 1.5 has a concave surface when seen from air of radius of curvature 12 cm. An object is situated in the denser medium at a distance of 9 cm from the pole locate the image due to refraction in air

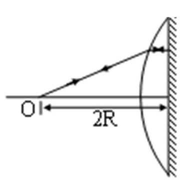
(A) A real image at 8 cm
(B) a virtual image at 8 cm
(C) A real image at 4.8 cm
(D) A virtual image at 4.8 cm

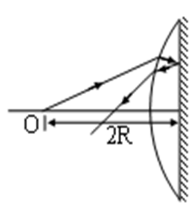
33. A point object O is placed at a distance of 20 cm from a convex lens of focal length 10 cm as shown in the figure. At what distance x from the lens should a convex mirror of focal length 60 cm, be placed so that final image coincide with the object?

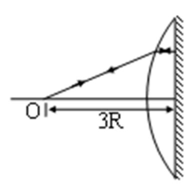


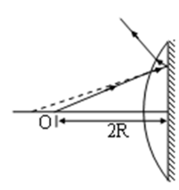
(A) 10 cm (B) 40 cm (C) 50cm (D) 20 cm

34. A thin plane convex glass lens ($\mu = 1.5$) has its plane surface silvered and R is the radius of curvature of the curved part. Then which of the following ray diagram is the correct representation for an object placed at O.

(A) 

(B) 

(C) 

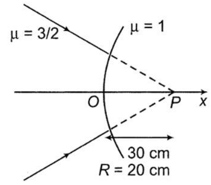
(D) 

35. A converging lens forms areal image I of an object on its principal axis. A rectangular slab of R.I = μ and thikness v is introduced between I & lens. The image I will move

(A) towards lens by $(\mu-1)x$ (B) away from lens by $\left(1-\frac{1}{\mu}\right)x$

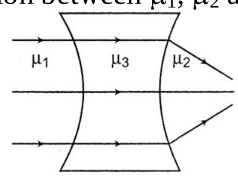
(C) away from lens by $(\mu - 1)x$ (D) towards the lens by $\left(1-\frac{1}{\mu}\right)x$

36. The image for the converging beam after refraction through the curved surface is formed at



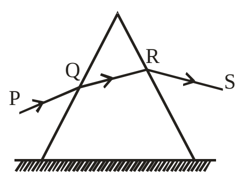
(A) $x = 40$ cm (B) $x = 40/3$ cm (C) $x = -40/3$ cm (D) $x = 20$ cm

37. For the figure shown, establish a relation between μ_1, μ_2 and μ_3



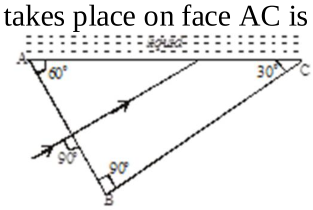
(A) $\mu_1<\mu_2<\mu_3$ (B) $\mu_3<\mu_2$; $\mu_3 = \mu_1$ (C) $\mu_3>\mu_2$; $\mu_3 = \mu_1$ (D) none of these

38. An equilateral prism is placed on a horizontal surface. A ray PQ is incident onto it. For minimum deviation



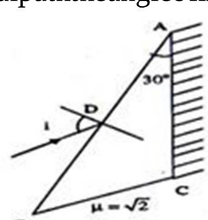
(A) PQ is horizontal (B) QR is horizontal
(C) RS is horizontal (D) Any one will be horizontal

39. Light is incident normally on face AB of a prism as shown in figure. A liquid of refractive index μ is placed on face AC of the prism. The prism is made of glass of refractive index 3/2. The limits of μ for which total internal reflection takes place on face AC is



(A) $\mu > \frac{\sqrt{3}}{2}$ (B) $\mu < \frac{3\sqrt{3}}{4}$ (C) $\mu > \sqrt{3}$ (D) $\mu < \frac{\sqrt{3}}{2}$

40. The prism shown in the figure has one side silvered. The angle of the prism is 30°and
$\mu = \sqrt{2}$. Iftheincidentrayretracesitsinitialpaththeangleofincidenceis

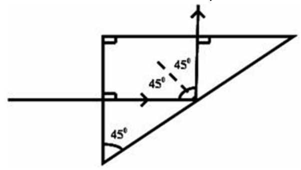


(A) 50^0 (B) 45^0 (C) 60^0 (D) 75^0

41. Light takes a time of t_1 seconds to travel a distance x cm in vacuum and takes t_2 sec to travel 10x cm in a medium. The critical angle of medium is

(A) $\sin^{-1}\left(\frac{10t_1}{t_2}\right)$ (B) $\sin^{-1}\left(\frac{t_2}{10t_1}\right)\frac{4}{3}$ (C) $\sin^{-1}\left(\frac{10t_2}{t_1}\right)\frac{3}{2}$ (D) $\sin^{-1}\left(\frac{t_1}{10t_2}\right)1.414$

42. A light ray is incident perpendicularly to one face of a 90^0 prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45^0, we conclude that the refractive index n



(A) $n > \frac{1}{\sqrt{2}}$ (B) $n > \sqrt{2}$ (C) $n < \frac{1}{\sqrt{2}}$ (D) $n < \sqrt{2}$

43. The velocities of light in two different media are $2\times10^8 ms^{-1}$ and $2.5\times10^8 ms^{-1}$ respectively, the critical angle for these media is

(A) $\sin^{-1}\left(\frac{1}{5}\right)$ (B) $\sin^{-1}\left(\frac{4}{5}\right)$ (C) $\sin^{-1}\left(\frac{1}{2}\right)$ (D) $\sin^{-1}\left(\frac{1}{4}\right)$

44. Two identical equi-convex lenses of focal length f, made of glass $\left(\mu_g = \frac{3}{2}\right)$ are kept in contact. The space between the two lenses is filled with water $\left(\mu_w = \frac{4}{3}\right)$. The focal length of the combination is

(A) f / 2 (B) 3f / 4 (C) f (D) 4f / 3

45. For an equilateral prism, it is observed that when a ray strikes grazingly at one face it emerges grazingly at the other. Its refractive index will be

(A) $\sqrt{3}$ (B) $\frac{2}{\sqrt{3}}$ (C) 2 (D) Data not sufficient

46. A biconvex lens of focal length f forms a circular image of radius r in focal plane. Then

(A) $\pi r^2 \propto f$ (B) $\pi r^2 \propto f^2$

(C) if lower half is converted by black sheet, then area of the image is equal to $\pi r^2/2$

(D) if f is doubled, intensity will increase

47. Two light sources are said to be coherent if they are obtained from

(A) Two independent point source emitting light of the same wavelength

(B) A single point source (C) Two different candles

(D) Two ordinary bulbs emitting light of different wavelength

48. Two source of same intensity interfere in phase at a point and produced resultant I. When one source is removed, the intensity at the point will be

(A) I (B) I/2 (C) I/4 (D) I/3

49. If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, the amplitude of the resulting wave will be about

(A) 7 (B) 6 (C) 5 (D) 3.5

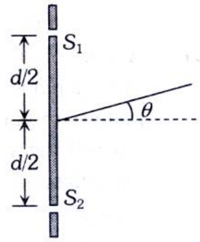
50. In Young's double slit experiment when wavelength used in 6000Å and the screen is 40 cm from the slits, the fringes are 0.012cm wide. What is the distance between the slits

(A) 0.024 cm (B) 2.4 cm (C) 0.24 cm (D) 0.2 cm

51. In Young's double slit experiment, white light is used. The separation between the slits is b. The screen is at a distance d(d >> b) from the slits. Some wavelengths are missing exactly in front of one slit. These wavelengths are

(A) $\lambda = \frac{b^2}{d}$ (B) $\lambda = \frac{2b^2}{d}$ (C) $\lambda = \frac{3b^2}{2d}$ (D) $\lambda = \frac{2b^2}{3d}$

52. In an interference arrangement similar to Young's double slit experiment, the slits S_1 and S_2 are illuminated with coherent microwave source each of frequency 10^6 Hz. The sources are synchronized to have zero phase difference. The slits are separated by distance d = 150 m. The intensity $I(\theta)$ is measured as a function of θ, where θ is defined as shown. If I_0 is maximum intensity, then $I(\theta)$ for $0 \leq \theta \leq 90^0$ is given by



(A) $I(\theta) = I_0$ for $\theta = 0^0$ (B) $I(\theta) = I_0/2$ for $\theta = 60^0$
(C) $I(\theta) = I_0/4$ for $\theta = 90^0$ (D) $I(\theta)$ is constant for all value of θ

53. If two waves, each of intensity I_0, having the same frequency but differing by a constant phase angle of 60^o, superpose, then the intensity of resultant wave is
(A) $2I_0$ (B) $\sqrt{3}I_0$ (C) $3I_0$ (D) $4I_0$

CHEMISTRY

Syllabus: ***SECOND YEAR PHYSICAL CHEMISTRY:– 1. SOLID STATE, 2. SOLUTIONS, 3. ELECTRO CHEMISTRY AND CHEMICAL KINETICS 4.SURFACE CHEMISTRY***

1. The aqueous solution that has the highest value of lowering of vapour pressure at a given temperature is
(A) 0.1 molal sodium phosphate (B) 0.1 molal barium chloride
(C) 0.1 molal sodium chloride (D) 0.1 molal glucose.
2. 12 g of urea is dissolved in 1 litre of water and 68.4 g of sucrose is dissolved in 1 litre of water. The lowring of vapour pressure of first reaction is
(A) equal to second (B) greater than second
(C) less than second (D) double that of second.
3. During depression in freezing point of a solution the following are in equilibrium
(A) liquid solvent, solid solvent (B) liquid solvent, solid solute
(C) liquid solute, solid solute (D) liquid solute, solid solvent
4. 0.6 g of a solute is dissolved in 0.1 litre of a solvent which develops an osmotic pressure of 1.23 atm at 27°C. The molecular mass of the substance is
(A) 149.5 g mol^{-1} (B) 120 g mol^{-1} (C) 430 g mol^{-1} (D) none of these
5. A 0.6 % (w/v) solution of urea (molecular weight = 60) would be isotonic with
(A) 0.1 M glucose solution (B) 0.1 M KCl solution
(C) 0.6 % (w/v) glucose solution (D) 0.6 % (w/v) KCl solution
6. If 0.1M solution of glucose and 0.1 M solution of urea are placed on two sides of the semi-permeable membrane to equal heights, then it will be correct to say that
(A) There will be no net movement across the membrane
(B) Glucose will flow towards urea solution
(C) Urea will flow from urea solution
(D) Water will flow from urea solution to glucose solution
7. The van't Hoff factor for 0.1 M $Ba(NO_3)_2$ solution is 2.74. The degree of dissociation is
(A) 91.3 % (B) 74 % (C) 87 % (D) 100 %

8. Pure benzene freezes at 5.3°C. A solution of 0.223 g of phenyl acetic acid ($C_6H_5CH_2COOH$) in 4.4 g of benzene $\left(K_f = 5.12 K\ kg\ mol^{-1}\right)$ freezes at 4.47°C. From this observation, one can conclude that
(A) Phenyl acetic acid exists as such in benzene
(B) Phenyl acetic acid undergoes partial ionization in benzene
(C) Phenyl acetic acid undergoes complete ionization in benzene
(D) Phenyl acetic acid dimerizes in benzene.

9. Osmotic pressure of urea solution at 10°C is 500 mm. The solution is diluted and temperature raised to 25°C till its osmotic pressure becomes 131.6 mm. The solution is diluted
(A) 3 times (B) 3.5 times (C) 4 times (D) 3.8 times

10. Calculate the weight of ethylene glycol (an effective antifreeze) that must be added to 25 litre water to protect its freezing at $-24°C \left(K_f = 1.86°C\ m^{-1}\right)$
(A) 20 kg (B) 322.5 kg (C) 200 kg (D) 32.25 kg

11. The percentage dissociation of a 0.011 m aqueous solution of $K_3\left[Fe\left(CN\right)_6\right]$ which freezes at –0.063°C is (K_f for water = 1.86)
(A)75% (B) 67 % (C) 33 % (D) 50 %

12. The freezing point of solution made by dissolving 1.1 g of $COCl_3.6NH_3$ (molecular weight = 267) in 100 g H_2O is –0.29°C. How many moles of solute particles exist in solution for each mole of solute introduced? (K_f = 1.86°C m^{-1})
(A) 2 (B) 3 (C) 4 (D) 1

13. At a particular temperature, the vapour pressure of two liquids A and B are 120 and 180 mm Hg respectively. If 2 moles of A and 3 moles of B are mixed to form an ideal solution. The vapour pressure of solution at same temperature will be (in mm Hg)
(A) 156 (B) 145 (C) 108 (D) 48

14. A 0.001 molal solution of $\left[Pt\left(NH_3\right)_4 Cl_4\right]$ in water had a freezing point depression of 0.0054°C. If K_f for water is 1.80°C m^{-1}, the correct formulation for the above molecule is
(A) $\left[Pt\left(NH_3\right)_4 Cl_3\right]Cl$ (B) $\left[Pt\left(NH_3\right)_4 Cl_2\right]Cl_2$
(C) $\left[Pt\left(NH_3\right)_4 Cl\right]Cl_3$ (D) $\left[Pt\left(NH_3\right)_4 Cl_4\right]$

15. For an ideal binary liquid solution with $P_A^\circ > P_B^\circ$, which relation between X_A (mole fraction of A in liquid phase) and Y_A(mole fraction of A in vapour phase) is correct?
(A) $Y_A < Y_B$ (B) $X_A > X_B$ (C) $\frac{Y_A}{Y_B} > \frac{X_A}{X_B}$ (D) $\frac{Y_A}{Y_B} < \frac{X_A}{X_B}$

16. Mole fraction of A vapours above the solution in mixture of A and B (X_A = 0.4) will be
[Given : = 100 mm Hg and = 200 mm Hg]
(A) 0.4 (B) 0.8 (C) 0.25 (D) none of these

17. The exact mathematical expression of Raoul's law is
(A) $\frac{P^0 - P_s}{P^0} = \frac{n}{N}$ (B) $\frac{P^0 - P_s}{P^0} = \frac{N}{n}$ (C) $\frac{P^0 - P_s}{P_s} = \frac{n}{N}$ (D) $\frac{P^0 - P_s}{P^0} = n \times N$

18. A mixture contains 1 mole of volatile liquid A (=100 mm Hg) and 3 moles of volatile liquid B (= 80 mm Hg). If solution behaves ideally, the total vapour pressure of the distillate is
(A) 85 mm Hg (B) 85.88 mm Hg (C) 90 mm Hg (D) 92 mm Hg

19. Which of the following aqueous solution will show maximum vapour pressure at 300 K?
(A) 1 M NaCl (B) 1 M $CaCl_2$ (C) 1 M $AlCl_3$ (D) 1 M $C_{12}H_{22}O_{11}$

20. The Van't Hoff factor for a dilute aqueous solution of glucose is
(A) zero (B) 1.0 (C) 1.5 (D) 2.0

21. The correct relationship between the boiling points of very dilute solution oif $AlCl_3$ (T_1K) and $CaCl_2$ (T_2K) having the same molar concentration is
(A) $T_1 = T_2$ (B) $T_1 > T_2$ (C) $T_2 > T_1$ (D) $T_2 < T_1$

22. A 0.001 molal solution of a complex $[MA_8]$ in water has the freezing point of –0.0054°C. Assuming 100% ionization of the complex salt and K_f for H_2O = 1.86 km^{-1}, write the correct representation for the complex
(A) $[MA_8]$ (B) $[MA_7]A$ (C) $[MA_6]A_2$ (D) $[MA_5]A_3$

23. The vapour pressure of a solution of a non-volatile electrolyte B in a solvent A is 95% of the vapour pressure of the solvent at the same temperature. If the molecular weight of the solvent is 0.3 times the molecular weight of solute, the weight ratio of the solvent and solute are
(A) 0.15 (B) 5.7 (C) 0.2 (D) 4.0

24. At a given temperature, total vapour pressure in Torr of a mixture of volatile components A and B is given by

$$P_{Total} = 120 - 75\,X_B$$

hence, vapour pressure of pure A and B respectively (in Torr) are
(A) 120, 75 (B) 120, 195 (C) 120, 45 (D) 75, 45

25. Assuming each salt to be 90 % dissociated, which of the following will have highest boiling point?
(A) Decimolar $Al_2(SO_4)_3$ (B) Decimolar $BaCl_2$
(C) Decimolar Na_2SO_4
(D) A solution obtained by mixing equal volumes of (2) and (3)

26. The vapour pressure of a solvent decreased by 10 mm of Hg when a non-volatile solute was added to the solvent. The mole fraction of solute in solution is 0.2, what would be mole fraction of the solvent if decrease in vapour pressure is 20 mm of Hg
(A) 0.2 (B) 0.4 (C) 0.6 (D) 0.8

27. The $2N$ aqueous solution of H_2SO_4 contains
(A) 49 gm of H_2SO_4 per litre of solution (B) 4.9 gm of H_2SO_4 per litre of solution
(C) 98 gm of H_2SO_4 per litre of solution (D) 9.8 gm of H_2SO_4 per litre of solution

28. The amount of $KMnO_4$ required to prepare 100 *ml* of $0.1N$ solution in alkaline medium is
(A) 1.58 gm (B) 3.16 gm (C) 0.52 gm (D) 0.31 gm

29. What weight of hydrated oxalic acid should be added for complete neutralisation of $100ml$ of $0.2\,N - NaOH$ solution ?
(A) $0.45\,g$ (B) $0.90\,g$ (C) $1.08\,g$ (D) $1.26\,g$

30. A $500g$ tooth paste sample has $0.2g$ fluoride concentration. What is the concentration of F in terms of *ppm* level
(A) 250 (B) 200 (C) 400 (D) 1000

31. To $5.85gm$ of $NaCl$ one kg of water is added to prepare of solution. What is the strength of $NaCl$ in this solution (mol. wt. of $NaCl = 58.5$)
(A) 0.1 Normal (B) 0.1 Molal (C) 0.1 Molar (D) 0.1 Formal

32. The degree of dissociation of $Ca(NO_3)_2$ in a dilute aqueous solution containing 14*g* of the salt per 200*g* of water $100°C$ is 70 percent. If the vapour pressure of water at $100°C$ is 760 *cm*. Calculate the vapour pressure of the solution
(A) 746.3 *mm* of *Hg* (B) 757.5 *mm* of *Hg* (C) 740.9 *mm* of *Hg* (D) 750 *mm* of *Hg*

33. The vapour pressure of pure benzene at a certain temperature is 200 *mm Hg*. At the same temperature the vapour pressure of a solution containing 2*g* of non-volatile non-electrolyte solid in 78*g* of benzene is 195 *mm Hg*. What is the molecular weight of solid

(A) 50 (B) 70 (C) 85 (D) 80

34. Which one of the following non-ideal solutions shows the negative deviation

(A) $CH_3COCH_3 + CS_2$ (B) $C_6H_6 + CH_3COCH_3$

(C) $CCl_4 + CHCl_3$ (D) $CH_3COCH_3 + CHCl_3$

35. The O.P. of equimolar solution of Urea, $BaCl_2$ and $AlCl_3$, will be in the order

(A) $AlCl_3 > BaCl_2 >$ Urea (B) $BaCl_2 > AlCl_3 >$ Urea

(C) Urea $> BaCl_2 > AlCl_3$ (D) $BaCl_2 >$ Urea $> AlCl_3$

36. The osmotic pressure of a 5% solution of cane sugar at $150^o C$ is (mol. wt. of cane sugar = 342)

(A) 4 atm (B) 3.4 atm (C) 5.07 atm (D) 2.45 atm

37. Which one of the following pairs of solutions can we expect to be isotonic at the same temperature

(A) $0.1M$ urea and $0.1M\ NaCl$ (B) $0.1M$ urea and $0.2M\ MgCl_2$

(C) $0.1M\ NaCl$ and $0.1M\ Na_2SO_4$ (D) $0.1M\ Ca(NO_3)_2$ and $0.1M\ Na_2SO_4$

38. Which of the following would have the highest osmotic pressure (assume that all salts are 90% dissociated)

(A) Decimolar aluminium sulphate (B) Decimolar barium chloride

(C) Decimolar sodium sulphate

(D) A solution obtained by mixing equal of (b) and (c) and filtering

39. Which solution will have the highest boiling point

(A) 1% solution of glucose in water (B) 1% solution of sodium chloride in water

(C) 1% solution of zinc sulphate in water (D) 1% solution of urea in water

40. The boiling point of a solution of 0.11 *gm* of a substance in 15 *gm* of ether was found to be $0.1^o C$ higher than that of the pure ether. The molecular weight of the substance will be $(K_b = 2.16)$

(A) 148 (B) 158 (C) 168 (D) 178

41. The boiling point of benzene is 353.23 *K*. When 1.80 *gm* of a nonvolatile solute was dissolved in 90 *gm* of benzene, the boiling point is raised to 354.11 *K*. the molar mass of the solute is $[K_b$ for benzene = 2.53 K mol$^{-1}]$

(A) $5.8\ g\ mol^{-1}$ (B) $0.58\ g\ mol^{-1}$ (C) $58\ g\ mol^{-1}$ (D) $0.88\ g\ mol^{-1}$

42. The boiling point of 0.1 moll aqueous solution of urea is $100.18^o C$ at 1 atm. The molal elevation constant of water is

(A) 1.8 (B) 0.18 (C) 18 (D) 18.6

43. The freezing point of a solution containing 4.8 *g* of a compound in 60 *g* of benzene is 4.48. What is the molar mass of the compound $(K_f = 5.1 km^{-1})$, (freezing point of benzene = $5.5^o C$)

(A) 100 (B) 200 (C) 300 (D) 400

44. When 0.01 mole of sugar is dissolved in $100g$ of a solvent, the depression in freezing point is 0.40^o. When 0.03 mole of glucose is dissolved in $50g$ of the same solvent, the depression in freezing point will be

(A) 0.60^o (B) 0.80^o (C) 1.60^o (D) 2.40^o

45. The freezing point of equimolal aqueous solution will be highest for

(A) $C_6H_5NH_3^+Cl^-$ (Aniline hydrochloride) (B) $Ca(NO_3)_2$

(C) $La(NO_3)_3$ (D) $C_6H_{12}O_6$ (Glucose)

46. The Van't Hoff factor of the compound $K_3Fe(CN)_6$ is

(A) 1 (B) 2 (C) 3 (D) 4

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MELUHA INTERNATIONAL SCHOOL

HYDERABAD

SR MPC JEE MAINS | UNIT – IV | Date: 03-05-2020

Time: | ASSIGNMENT – 2 | Max. Marks:

MATHS

1) **A**	2) **A**	3) **B**	4) **A**	5) **C**	6) **C**	7) **A**	8) **B**	9) **B**	10) **A**
11) **B**	12) **A**	13) **C**	14) **A**	15) **B**	16) **A**	17) **A**	18) **A**	19) **B**	20) **D**
21) **B**	22) **C**	23) **B**	24) **B**	25) **B**	26) **D**	27) **B**	28) **A**	29) **A**	30) **A**
31) **C**	32) **A**	33) **A**	34) **A**	35) **A**	36) **C**	37) **A**	38) **B**	39) **A**	40) **A**
41) **C**	42) **B**	43) **C**	44) **D**	45) **A**	46) **A**	47) **B**	48) **B**	49) **B**	50) **A**
51) **C**	52) **C**	53) **D**	54) **C**	55) **D**	56) **B**	57) **D**	58) **B**	59) **A**	60) **A**
61) **B**	62) **C**	63) **A**	64) **A**	65) **A**	66) **A**	67) **A**	68) **B**	69) **C**	70) **A**
71) **C**	72) **C**	73) **A**	74) **A**	75) **A**	76) **B**	77) **C**	78) **A**	79) **A**	80) **D**
81) **C**	82) **D**	83) **A**	84) **B**	85) **B**	86) **D**	87) **D**	88) **B**	89) **A**	90) **B**
91) **B**	92) **A**	93) **D**	94) **C**	95) **A**	96) **C**	97) **A**	98) **B**	99) **A**	100) **A**
101) **B**	102) **B**	103) **C**	104) **C**	105) **A**	106) **C**	107) **A**	108) **C**	109) **D**	110) **D**
111) **A**	112) **D**	113) **B**	114) **D**	115) **B**	116) **B**	117) **A**	118) **A**	119) **C**	120) **A**
121) **C**	122) **B**	123) **C**	124) **D**	125) **C**					

PHYSICS

1) **D**	2) **B**	3) **C**	4) **B**	5) **B**	6) **D**	7) **A**	8) **B**	9) **B**	10) **A**
11) **C**	12) **C**	13) **B**	14) **D**	15) **A**	16) **D**	17) **C**	18) **C**	19) **D**	20) **B**
21) **D**	22) **C**	23) **A**	24) **D**	25) **D**	26) **A**	27) **B**	28) **B**	29) **C**	30) **C**
31) **B**	32) **D**	33) **D**	34) **A**	35) **B**	36) **A**	37) **B**	38) **B**	39) **B**	40) **B**
41) **A**	42) **B**	43) **B**	44) **B**	45) **C**	46) **B**	47) **B**	48) **C**	49) **B**	50) **D**
51) **A**	52) **A**	53) **C**							

CHEMISTRY

1) **A**	2) **A**	3) **A**	4) **B**	5) **A**	6) **A**	7) **C**	8) **D**	9) **C**	10) **A**
11) **B**	12) **C**	13) **A**	14) **B**	15) **C**	16) **C**	17) **C**	18) **B**	19) **D**	20) **B**
21) **B**	22) **C**	23) **B**	24) **C**	25) **A**	26) **C**	27) **C**	28) **A**	29) **D**	30) **C**
31) **B**	32) **A**	33) **D**	34) **D**	35) **A**	36) **C**	37) **D**	38) **A**	39) **B**	40) **B**
41) **C**	42) **A**	43) **D**	44) **D**	45) **D**	46) **D**				

HINTS & SOLUTIONS

MATHS

1. (A) $f'(x)=\frac{1}{x^2+2}\times 2x=0 \quad x=0$

2. (A) Apply Lagrange's therom $f'(c)=\frac{f(q)-f(p)}{q-p}$

3. (B) is increasing $\Rightarrow f'(x)\geq 0 \Rightarrow 2x+k\geq 0$

$\Rightarrow x\geq \frac{-k}{2}$ since $1\leq x\leq 2 \Rightarrow 1\geq -\frac{k}{2}\Rightarrow k\geq -2$

$\therefore$ least value of k is -2.

44. (A) Given $f(x)=x^3-3x^2-9x+20$

$\Rightarrow f'(x)=3x^2-6x-9$

$\Rightarrow f'(x)=3(x-3)(x+1)$

$f'(x)>0 \Rightarrow x\in(-\propto,-1)\cup(3,\propto)$

$f'(x)>0 \Rightarrow x\in(-1,3)$

Thus, $f(x)$ is strictly increasing for

$x\in(-\propto,-1)\cup(3,\propto)$ and strictly decreasing for $x\in(-1,3)$

5. (C) $f(x)=(x-2)^{\frac{2}{3}}(2x+1);\ f'(x)=2\left[\frac{5x-5}{(x-2)^{\frac{1}{3}}}\right]$

$f'(x)=0 \Rightarrow x=1;\ f^1(x)$ does not exist at x = 2

$\therefore$ x = 1 and x = 2 are two critical points

6. (C) $f(x)=\sin x \Rightarrow f'(x)=\cos x \Rightarrow f'(x)=0$

$\Rightarrow x=(2n+1)\frac{\pi}{2}$. Therefore number of stationary points of $f(x)$ in $[0,2\pi]$ is 2.

7. (A) $AM>GM$

8. (B) $f'(x)=-\frac{a}{x^2}+2x$ since $f(x)$ has local maximum at $x=-3 \Rightarrow f'(-3)=0$ and $f^{11}(-3)<0$

For $f^1(-3)=0 \Rightarrow a=-54,\ x=-3, a=-54$

Now, $f^{11}(-3)<0$, Hence, $a=-54$

9. (B) $f^1(x)=4(x-1)^3$ $\qquad f^{11}(x)=12(x-1)^2$

$f^{111}(x)=24(x-1)$ $\qquad f^{iv}(x)=24$

$f^1(1)=0, f^{11}(1)=0,$ $\qquad f^{iv}(1)\neq 0$

Therefore $n=iv$ is even and $f^{iv}(1)=24>0$

Therefore $f(x)$ has local minimum at x = 1.

10. (A) $f^1(x)=6(x-1)(x-2)$

$f^1(x)=0 \Rightarrow x=1, x=2$

$\Rightarrow f(0)=6, f(1)=11, f(2)=10$

Therefore global maximum $M_1 = \max\{f(0), f(1), f(2)\} = 11$

Global minimum

$M_2 = \max\{f(0), f(1), f(2)\} = 6$

11. (B)

$f'(x) = 2(x-1)(x+2) + (x+2)^2 = 3x^2 + 6x$

$f'(x) = 0 \Rightarrow 0, -2$

$f(-2) = (-2-1)(-2+2)^2 = 0$ [Maximum value]

and $f(0) = (0-1)(0+2)^2 = -4$ [Minimum value].

12. (A) Find $\frac{dy}{dx}$ to the two curves at (1, 1) they are m_1 and m_2. Then $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

13. (C) Obviously here $\cos 3x$ is not decreasing in $\left(0, \frac{\pi}{2}\right)$

Because $\frac{d}{dx}\cos 3x = -3\sin 3x$

But at $x = 75^0, -3\sin 3x > 0.$ Hence the result.

14. (A) $\frac{dy}{dx} = \frac{2t-1}{2t}$ tangent is perpendicular to x- axis $m = \infty$

15. (B) Here $\frac{f(b) - f(a)}{b-a} = f'(c)$

$\Rightarrow \frac{e^b - e^a}{b-a} = f'(c) \Rightarrow \frac{e-1}{1-0} = e \Rightarrow c = \log(e-1).$

16. (A) $f'(x_1) = \frac{-1}{x_1^2}$

$\therefore \frac{-1}{x_1^2} = \frac{\frac{1}{b} - \frac{1}{a}}{b-a} = -\frac{1}{ab} \Rightarrow x_1 = \sqrt{ab}.$

17. (A) $f^1(x) = 0 \Rightarrow (2x+1)(1-x+x^2) - (2x-1)(1+x+x^2) = 0$

$\Rightarrow x = \pm 1 \,\&\, f_{11}(-1) > 0$

18. (A) $f^1(x) = P - \frac{qn^2}{x^2}$

$f^1(x) = 0 \Rightarrow x = \pm n\sqrt{\frac{q}{p}}$

$f^{11}(x) > 0 \quad for\ x = n\sqrt{\frac{q}{p}}$

$\therefore$ Minimum value $= 2n\sqrt{pq}$

19. (B) Max value of $f(x) = 81$

$f(x) = (x+1)^{1/3} - (x-1)^{1/3}$

$f'(x) = \frac{(x-1)^{\frac{2}{3}} - (x+1)^{\frac{2}{3}}}{3(x^2-1)^{\frac{2}{3}}}$

clearly $f^1(x)$ does not exists at $x = \pm 1.$

Now $f^1(x) = 0$

then $(x-1)^{2/3}=(x+1)^{2/3} \Rightarrow x=0$

clearly $f^1(x)\neq 0$ for any other value of $x\in[0,1]$

The value of $f(x)$ at $x=0$ is 2

Hence the greatest value of $f(x)$ is 2

20. (D) $\frac{dy}{dx}=\frac{(2x-2)}{2y}=0$

$\Rightarrow$ $x=1, 1+y^2-2-3=0 \Rightarrow y=\pm 2$, points are (1, 2), (1, –2).

21. (B) Using formula $f'(c)=\frac{f(b)-f(a)}{b-a}$.72. (C)

$f'(c)=0$

$c^2(c-2)+(c-2)^2=0$

$c=2, \frac{2}{3}$

$\therefore$ $c=\frac{2}{3}(c\neq 2)$

23. (B) $m=\left(\frac{dy}{dx}\right)_{(1,1)}$

24. (B) F(x) = [x] is discontinuous function in [–1, 1].

25. (B) $A=B=\frac{\pi}{6}$, then $\tan A.\tan B=\frac{1}{3}$.

26. (D) $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

27. (B) Put x = 0 in curve and find point at that point find the tangent

28. (A) $f'(x)=0 \Rightarrow x=+1$

$f''(x)=6x$

$\Rightarrow f''(-1)<0$

$\Rightarrow f(\theta)=2$, $f(-1)4, f(2)=4$

29. (A) $V=\frac{1}{3}\pi r^2h=\frac{1}{3}\pi\left(l^2-h^2\right)h=\frac{1}{3}\pi\left(l^2h-h^3\right)$

$\Rightarrow \frac{dv}{dt}=\frac{1}{3}\pi\left(l^2-3h^2\right)\frac{dh}{dt}$

30. (A) We have, $y^2=18x$

$\Rightarrow 2y\frac{dy}{dt}=18\frac{dx}{dt}$

$\Rightarrow 2y\times 2\frac{dx}{dt}=18\frac{dx}{dt}$ $\left[\because \frac{dy}{dt}=2\frac{dx}{dt}\right]$

$\Rightarrow 4y=18 \Rightarrow y=\frac{9}{2}$

When $y=\frac{9}{2}$, we have

$\left(\frac{9}{2}\right)^2=18x \Rightarrow x=\frac{9}{8}$

Hence, the required point is (9/8, 9/2)

31. (C) Let $y = x^{3000}$, x = 1 and x + Δx = 1.0002
Then, Δx = 1.0002 – 1 = 0.0002
Also, y = 1 when x = 1
Now,
$y = x^{3000}$

$$\Rightarrow \frac{dy}{dx} = 3000\, x^{2999} \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{x=1} = 3000$$

$$\therefore \Delta y = \frac{dy}{dx}\Delta x$$

$$\Rightarrow \Delta y = 3000 \times 0.0002 = \frac{6}{10} = 0.6$$

Hence, $(1.0002)^{3000} = y + \Delta y = 1 + 0.6 = 1.6$

32. (A) Let r be the radius, S the surface area and V the volume of the sphere. Then,

$$S = 4\pi r^2 \text{ and } V = \frac{4}{3}\pi r^3$$

Let Δr, ΔS and ΔV be the errors in r, S and V respectively. Then,

$$\Delta S = \frac{dS}{dr}\Delta r \text{ and } \Delta V = \frac{dV}{dr}\Delta r$$

$$\Rightarrow \Delta S = 8\pi r\, \Delta r \text{ and } \Delta V = 4\pi r^2\, \Delta r$$

$$\Rightarrow \frac{\Delta S}{S} = \frac{8\pi r}{4\pi r^2}\Delta r \text{ and } \frac{\Delta V}{V} = \frac{4\pi r^2\, \Delta r}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \frac{\Delta S}{S}\times 100 = 2\left(\frac{\Delta r}{r}\times 100\right) \text{ and } \frac{\Delta V}{V}\times 100 = 3\left(\frac{\Delta r}{r}\times 100\right)$$

$$\Rightarrow \alpha = 2\left(\frac{\Delta r}{r}\times 100\right) \text{ and } \frac{\Delta V}{V}\times 100 = 3\left(\frac{\Delta r}{r}\times 100\right) \left[\because \frac{\Delta S}{S}\times 100 = \alpha\ (given)\right]$$

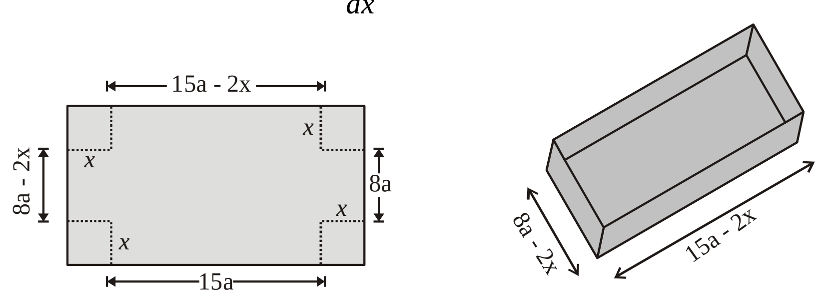
$$\Rightarrow \frac{\Delta V}{V}\times 100 = 3\left(\frac{\alpha}{2}\right) = \frac{3}{2}\alpha$$

33. (A)
Let the sides of the rectangular sheet be 15a and 8a units and the length of the side of each square to be cut from each corner of the sheet be x units. Then the dimension of the box are
Length = $15a - 2x$, Breadth = $8a - 2x$, Depth = x
Clearly, Length > 0, Breadth > 0 and Depth > 0. Therefore, $0 < x < 4a$.
Let V be the volume of the box. Then,

$$V = (15a - 2x)(8a - 2x)\,x = 120a^2x - 46ax^2 + 4x^3$$

$$\therefore \frac{dV}{dx} = 120a^2 - 92ax + 12x^2$$

The critical numbers of V are given by $\frac{dV}{dx} = 0$.



$\therefore\ \frac{dV}{dx}=0.$

$\Rightarrow 120a^2-92ax+12x^2=0$

$\Rightarrow 30a^2-23ax+3x^2=0$

$\Rightarrow (6a-x)(5a-3x)=0$

$\Rightarrow 5a-3x=0\ \left[\because\ 0<x<4a\quad \therefore\ 6a-x\neq 0\right]$

$\Rightarrow x=\frac{5a}{3}$

When $x=\frac{5a}{3}\frac{d^2V}{dx^2}=-92a+40a=-52a<0$

It is given that the total area of all squares cut from each corner of the sheet is 100 sq. units.

Therefore, $4x^2 = 100 \Rightarrow 4\left(\frac{5a}{3}\right)^2=100\ \Rightarrow\ a^2=9\ \Rightarrow\ a=3$

Hence, the dimensions of the sheet are 8a = 24 and 15a = 45 units

34. (A) For decreasing $f^1(x)<0, \cos x<a\ \Rightarrow\ a>1$

35. (A) at x = 45 f takes maximum value

36. (C) Length of the sub-normal = $|y_1 m|$.

37. (A) Length of sub-normal = $|y_1 m|$.

38. (B) We have,

$f(x)=x^{3/2}(3x-10).x\geq 0$

$\Rightarrow f'(x)=\frac{3}{2}x^{1/2}(3x-10)+x^{3/2}\times 3=\frac{15}{2}x^{1/2}(x-2)$

For f(x) to be increasing, we must have

$f'(x)\geq 0\ \Rightarrow\ \frac{15}{2}x^{1/2}(x-2)\geq 0\ \Rightarrow\ x\geq 2\ \Rightarrow\ x\in[2,\infty)$

39. (A)

We have, $f(x)=\log(\sin x+\cos x)$

$\therefore\ f'(x)=\frac{\cos x-\sin x}{\sin x+\cos x}=\frac{1-\tan x}{1+\tan x}=\tan\left(\frac{\pi}{4}-x\right)$

For $f(x)$ to be increasing, we must have

$f'(x)>0$

$\Rightarrow \tan\left(\frac{\pi}{4}-x\right)>0$

$\Rightarrow 0<\frac{\pi}{4}-x<\frac{\pi}{2}\ \Rightarrow\ -\frac{\pi}{4}<-x<\frac{\pi}{4}$

$\Rightarrow -\frac{\pi}{4}<x<\frac{\pi}{4}\ \Rightarrow\ x\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$

40. (A)

$f(x)=x^2=3x+3\ \Rightarrow\ f^{-1}(x)=2x-3\ \Rightarrow\ f(x)$ is said to be maximum or minimum

$f^1(x)=0\ \Rightarrow\ x=\frac{3}{2}$. Minimum $\left\{f(-3),\ f\left(\frac{3}{2}\right)\right\}=\left\{21,\frac{3}{4}\right\}$

41. (C) $\cos^4 x=\left(1-\sin^2 x\right)^2$

42. (B) $I=\int\left[\left(\sec x+\tan x\right)^{2}\right]^{\frac{1}{2}}dx$

43. (C) Put $1+\frac{1}{x^{2}}=t^{2}$ and use by parts

44. (D)Divide with $x^{2n}G.I=\int\frac{mx^{m-1}-\frac{n}{x^{n+1}}}{\left(x^{m}+\frac{1}{x^{n}}\right)^{2}}dx$ and

Put $x^{m}+\frac{1}{x^{n}}=t.$

45. (A) Put $1+x^{3}=t^{2}$

46. (A) $\frac{\sin 2x}{\sin^{4}x+\cos^{4}x}=\frac{2\tan x\sec^{2}x}{1+\tan^{4}x}$

47. (B) $GI=\int x\left(x+\sqrt{x^{2}-1}\right)dx$

$=\int x^{2}dx+\frac{1}{2}\int\left(x^{2}-1\right)^{\frac{1}{2}}2xdx$

48. (B) Put $\tan^{-1}\left(\frac{x^{2}+1}{x}\right)=t$

49. (B) Put $\cos 3x=4\cos^{3}x-3\cos x$

50. (A) Put $l^{r}\left(x\right)=t$

51. (C) Put $\cos^{\frac{3}{2}}x=t$

52. (C) $\sin^{6}x+\cos^{6}x=\sin^{4}x-\sin^{2}x\cos^{2}x+\cos^{4}x$

53. (D) $I=\int\frac{2\cos x-3\sin x}{6\cos x+4\sin x}dx$

Put $t=6\cos x+4\sin x$

$dt=\left(-6\sin x+4\cos x\right)dx$

$=2\left(2\cos x-3\sin x\right)dx$

$I=\int\frac{dt}{2t}=\frac{1}{2}\ln t+c$

$=\frac{1}{2}\ln\left(6\cos x+4\sin x\right)+c$

54. (C) $I=\int\frac{\left(\sin^{2}x+\cos^{2}x\right)^{2}}{\cos^{4}x\sin^{2}x}dx$

55. (D) $\int\frac{\sin^{8}c-\cos^{8}x}{1-2\sin^{2}x\cos^{2}x}dx=\int\left(\sin^{4}x-\cos^{4}x\right)dx=-\int\cos 2xdx$

56. (C) $\frac{\cos 2x-\sin 2x}{\cos 2x+\sin 2x}=\frac{1-\tan 2x}{1+\tan 2x}=\tan\left(\frac{\pi}{4}-2x\right)$

57. (D) Put $1+x^{n}=t^{2}$

58. (B) Put $\cos x=t$

59. (A) Put $\cos x=t$

60. (A) Apply the formula, $\int\frac{f'(x)}{f(x)}dx$

61. (B) Apply the formula, $\int\left(f(x)\right)^{n}f'(x)dx$

62. (C) Apply the formula, $\int \frac{f'(x)}{f(x)} dx$

63. (A) Put $x^3 = t$ and apply the formula, $\int \frac{1}{\sqrt{1-x^2}} dx$.

64. (A) Use, $\int e^x u\, dx$

65. (A) $\frac{x+\sin x}{1+\cos x} = \frac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$

66. (A) $x+4 = (x+6)-2$

67. (A) Put, $x = \sin\theta$

68. (B) $\frac{ax+b}{cx+d} = \frac{a}{c} + \frac{bc-ad}{c}\frac{1}{cx+d}$

69. (C) Apply the formula, $I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}.I_{n-2}$

70. (A) $I = \int x^{\log_e 5} dx$ A

71. (C) $U = \log\left(\tan\frac{x}{2}\right), V = \cos x$ or use integration by parts

72. (C) $GI = x^2\left(\frac{-1}{x\tan x+1}\right) - \int \frac{2x(-1)}{x\tan x+1} dx$

$= \frac{-x}{x\tan x+1} + 2\int \frac{x\cos x}{x\sin x+\cos x} dx$

73. (A) Put $\log x = t$

74. $I = \int \frac{dx}{x(x^n+1)} = \int \frac{x^{n-1}}{x^n(x^n+1)} dx$

Putting $x^n = t$ so that $n x^{n-1} dx = dt$

$\Rightarrow x^{n-1}dx = \frac{1}{n}dt$

$\therefore I = \int \frac{\frac{1}{n}dt}{t(t+1)} = \frac{1}{n}\int\left(\frac{1}{t} - \frac{1}{t+1}\right)dt$

$= \frac{1}{n}(\log t - \log(t+1)) + c$

$= \frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + c$

75. $I = \int\left(\frac{x+2}{x+4}\right)^2 e^x dx = \int e^x\left[\frac{x^2+4x+4}{(x+4)^2}\right]dx$

$\Rightarrow \quad I = \int e^x\left[\frac{x(x+4)}{(x+4)^2} + \frac{4}{(x+4)^2}\right]dx$

$= e^x\left[\frac{x}{x+4} + \frac{4}{(x+4)^2}\right]dx = e^x\left(\frac{x}{x+4}\right) + c$

76. $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx$

$$=\int\frac{2\cos^2 2x}{\cos^2 x-\sin^2 x}\sin x\cos x dx$$
$$=\int\frac{2\cos^2 2x}{\cos^2 x-\sin^2 x}\sin x\cos x dx$$
$$=\int\cos 2x\sin 2x dx$$
$$=\frac{1}{2}\int\sin 4x dx=-\frac{1}{8}\cos 4x+c$$

77. $\sin^3 x\sin(x+\alpha)$
$$=\sin^3 x(\sin x\cos\alpha+\cos x\sin\alpha)$$
$$=\sin^4 x(\cos\alpha+\cot x\sin\alpha)$$
$$I=\int\frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}dx=\int\frac{1}{\sin^2 x\sqrt{\cos\alpha+\cot x\sin\alpha}}dx=\int\frac{\text{cosec}^2 x}{\sqrt{\cos\alpha+\cot x\sin\alpha}}dx$$
Putting $\cos\alpha+\cot x\sin\alpha=t$ and $-\text{cosec}^2\sin\alpha dx=dt$, we have
$$I=\int-\frac{1}{\sin\alpha\sqrt{t}}dt=-\frac{1}{\sin\alpha}\int t^{-1/2}dt=-\frac{1}{\sin\alpha}\left(\frac{t^{1/2}}{1/2}\right)+C$$
$$\Rightarrow\quad I=-2\,\text{cosec}\alpha\sqrt{t}+C=-2\,\text{cosec}\alpha(\cos\alpha+\cot x\sin\alpha)^{1/2}+c$$

78. (A) $x+4=(x+6)-2$

79. (A) Put, $x=\sin\theta$

80. $I=\int\frac{dx}{\sqrt{\sin^3 x\cos^5 x}}=\int\frac{dx}{\sqrt{\frac{\sin^3 x}{\cos^3 x}\cos^8 x}}=\int\frac{\sec^4 x}{\sqrt{\tan^3 x}}dx=\int\frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan^3 x}}dx$

Let t = tanx $\Rightarrow$ dt = $\sec^2 x$ dx
$$\Rightarrow\quad I=\int\frac{1+t^2}{t^{3/2}}dt=\int\left(t^{-3/2}+t^{1/2}\right)dt=-2t^{-1/2}+\frac{2}{3}t^{3/2}+c=-2\sqrt{\cot x}+\frac{2}{3}\sqrt{\tan^3 x}+c$$
$$\Rightarrow\quad a=-2,\ b=\frac{2}{3}$$

81. $I=\int\frac{\ln(\tan x)}{\sin x\cos x}dx$, let t = ln (tanx)
$$\Rightarrow\quad\frac{dt}{dx}=\frac{\sec^2 x}{\tan x}\Rightarrow dt=\frac{dx}{\sin x\cos x}$$
$$\Rightarrow\quad I=\int t dt=\frac{1}{2}t^2+c=\frac{1}{2}(\ln(\tan x))^2+c$$

82. $I=\int\frac{\sqrt{x-1}}{x\sqrt{x+1}}dx$
$$=\int\frac{x-1}{x\sqrt{x^2-1}}dx=\int\frac{dx}{\sqrt{x^2-1}}-\int\frac{dx}{x\sqrt{x^2-1}}$$
$$=\ln\left|x+\sqrt{x^2-1}\right|-\sec^{-1}x+c$$

83. $A=\int\sqrt{e^x-1}dx$

Let $e^x-1=t^2\Rightarrow e^x dx=2t\,dt\Rightarrow dx=\frac{2t}{t^2+1}dt$
$$\Rightarrow\quad I=\int t\frac{2t}{t^2+1}dt=\int\frac{2t^2}{t^2+1}dt$$
$$=\int\frac{2(t^2+1)-2}{t^2+1}dt=\int 2\,dt-\int\frac{2dt}{t^2+1}$$

$= 2t - 2\tan^{-1} t + c$

$= 2\sqrt{e^x - 1} - 2\tan^{-1}\sqrt{e^x - 1} + C$

84. (B)

Let $e^x = t$

$\int \frac{(t^2+1)}{t^4 - t^2 + 1} dt$

$\int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2} - 1} dt$

$t - \frac{1}{t} = u$

$\left(1 + \frac{1}{t^2}\right) dt = du$

$\int \frac{du}{u^2 + 1} = \tan^{-1} u + c$

$\tan^{-1}\left(e^x - e^{-x}\right) + c$

85. (B) $\frac{1}{(x^2+1)(x^2+4)} dx$

$\frac{1}{3}\int \frac{(x^2+4)-(x^2+1)}{(x^2+1)(x^2+4)} dx$

$\frac{1}{3}\int \frac{1}{x^2+1} dx - \frac{1}{3}\int \frac{1}{x^2+4} dx$

$\frac{1}{3}\tan^{-1} x - \frac{1}{6}\tan^{-1}\frac{x}{2} + c$

86. (D) $\frac{1}{3}\int \frac{(x^2+4)+(2x^2+2)}{(x^2+1)(x^2+4)} dx$

87. (D) Put $x + 2 = y$

88. (B) $I = \int \left[1 - \frac{x+1}{x(x^2+1)}\right] dx$

$I = \int \left[1 - \frac{1}{x^2+1} - \left(\frac{1}{x} - \frac{x}{x^2+1}\right)\right] dx$

89. (A) Put $x + 1 = t^3$

90. (B) $\frac{ax+b}{cx+d} = \frac{a}{c} + \frac{bc-ad}{c}\frac{1}{cx+d}$

91. (B) Multiplying and dividing with $\sqrt{2}$

92. (A) Put $\frac{1}{x} = \cos\theta$

93. (D) $I = \frac{1}{4}\int \frac{4x^3}{(x^4+1)} + c = \frac{1}{4}\ln(x^4+1) + c$

94. (C) Apply the formula, $I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1}.I_{n-2}$

95. (A) $\frac{1}{2}\int \frac{dx}{\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x}$

$\frac{1}{2}\int \frac{dx}{\cos\left(x-\frac{\pi}{3}\right)}$ $\qquad$ $\frac{1}{2}\sec\left(x-\frac{\pi}{3}\right)dx$

$\frac{1}{2}\log\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$

96. (C)

$\int \frac{\sin\left(\left(x-\frac{\pi}{4}\right)+\frac{\pi}{4}\right)}{\sin\left(x-\frac{\pi}{4}\right)}dx$

$\int \frac{\sin\left(x-\frac{\pi}{4}\right)\cos\frac{\pi}{4}}{\sin\left(x-\frac{\pi}{4}\right)}+\int \frac{\sin\frac{\pi}{4}\cos\left(x-\frac{\pi}{4}\right)}{\sin\left(x-\frac{\pi}{4}\right)}dx$

$x+\log\left|\sin\left(x-\frac{\pi}{4}\right)\right|+c$

97. (A)

$\int x^5 f\left(x^3\right)dx$ $\qquad$ $x^3=t \;\Rightarrow\; 3x^2dx=dt$

$\int tf(t)\frac{dt}{3}=\frac{1}{3}\left[t\int f(t)dt-\int\psi(t)dt\right]$ $\qquad$ $=\frac{1}{3}\left[x^3\psi\left(x^3\right)-\int\psi\left(x^3\right)3x^2dx\right]+C$

98. (B)

$I=\int 1+x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$

$=\int e^{x+\frac{1}{x}}dx+\int x.\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$ $\qquad$ (using by parts)

$=\int e^{x+\frac{1}{x}}dx+x.e^{x+\frac{1}{x}}-\int 1.e^{x+\frac{1}{x}}dx$ $\qquad$ $=xe^{x+\frac{1}{x}}+c$

99. (A)

$\int e^x\left[\frac{x^2-1+2}{(x+1)^2}\right]dx$

$=\int e^x\left[\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right]dx$ (using $\int e^x\left(f(x)+f'(x)\right)dx$) = e^x f(x) + c

$=e^x\frac{x-1}{x+1}+c$

100. (A) $\int e^x\left[\frac{x\left(x^2+1\right)}{\left(x^2+1\right)\sqrt{x^2+1}}+\frac{1}{\left(x^2+1\right)^{3/2}}\right]dx$ (using $\int e^x\left(f(x)+f'(x)\right)dx$) = e^x f(x) + c

$=e^x\frac{x}{\sqrt{x^2+1}}+c$

101. (B) Using formula, $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$

102. (B) Using formula, $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

104. (C) $\int_0^n [x]dx = 0+1+2....+(n-1) = \frac{n(n-1)}{2}$

105. (A) $\int_0^a \sqrt{\frac{a+x}{a-x}}\,dx = \int_0^a \frac{a+x}{\sqrt{a^2-x^2}}dx$

106. (C) $\int_0^1 \frac{dx}{e^x+e^{-x}} = \int_0^1 \frac{e^x dx}{(e^x)^2+1}$, put $e^x = t$

107. (A) $\int_1^2 \log x\,dx = [x\log x - x]_1^2 = 2\log 2 - 1$

108. (C) $\int e^x[f(x)+f'(x)]dx = e^x f(x)$

109. (D) $\int_b^a \sqrt{(x-a)(b-x)}dx = \frac{\pi}{8}(b-a)^2$

110. (D) Using formula, $\int_0^{\pi/2} \frac{f(\cot x)}{f(\cot x)+f(\tan x)}dx = \frac{\pi}{4}$ g

111. (A)

$I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ ... (i)

$= \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx$ ... (ii)

Adding (i) and (ii)

112. (D) Given function is odd.

113. (B) We know that if f(t) is an odd function, then

$f(x) = \int_0^x f(t)\,dt$ is an even function

Since, $1\log\left(t+\sqrt{1+t^2}\right)$ is odd

$\therefore$ the given integral is even.

114. (D) $\int_{mT}^{nT} f(x)dx = (n-m)\int_0^T f(x)dx$

115. (B) Even function.

116. (B) Apply the formula, $\int \left(f(x)\right)^n f'(x)\,dx$.

117. (A) Put x^3 = t and apply the formula, $\int \frac{1}{\sqrt{1-x^2}}dx$.

118. (A) Use, $\int e^x u\,dx$

119. (C) $I = \frac{1}{\log_e 10}\int \log x\,dx$

120. (A) Put, U = x, V = $\tan x \sec^2 x$.

121. (C) $\int e^x[f(x)-f'(x)]dx = \phi(x)$ … (i)

$\int e^x[f(x)+f'(x)]dx = e^x f(x)$ … (ii)

Adding (i) and (ii).

122. (B) $\frac{1-\sin x}{1-\cos x} = \frac{1}{2}\operatorname{cosec}^2\frac{x}{2} - \cot\frac{x}{2}$

123. $\int \frac{dx}{(x+2)(x^2+1)} = a\ln(1+x^2) + b\tan^{-1}x + \frac{1}{5}\ln|x+2| + C$

Differentiating both sides, we get

$$\frac{1}{(x+1)(x^2+1)} = \frac{2ax}{(1+x^2)} + \frac{b}{(1+x^2)} + \frac{1}{5(x+2)}$$

$$\Rightarrow \quad \frac{1}{(x+1)(x^2+1)} = \frac{(x+2)(5b+10ax)+1+x^2}{5(1+x^2)(x+2)}$$

$$\Rightarrow \quad 5 = (1+x^2) + 5(b+2ax)(x+2)$$

Comparing the like powers of x in both sides, we get

1+10 a = 0, b +4a = 0, 10b+1 = 5

a = -1/10, b = 2/5

124. (A) $\frac{x+\sin x}{1+\cos x} = \frac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}$

125. (C) Put, $U = x^2$, $V = 2^{3x}$

PHYSICS

1. $\cos A = \frac{1}{8} \Rightarrow 1 - 2\sin^2\frac{A}{2} = \frac{1}{8} \Rightarrow \sin\frac{A}{2} = \frac{\sqrt{7}}{4}$

Now required $\mu = \operatorname{cosec}\frac{A}{2} = \frac{4}{\sqrt{7}}$

$\therefore k = 4$

6. $\mu_1 \sin i = \mu_4 \sin r$

For AB and CD will be parallel then i = r

Hence, $\mu_1 = \mu_4$

10.

$$\sin\theta_C = \sin\theta = \frac{\mu_R}{\mu_D} = \frac{c/v_B}{c/v_A} = \frac{v_A}{v_B} = \frac{v}{v_B}$$

$\therefore \quad v_B = \frac{v}{\sin\theta}$

$\therefore k = 4$

13. Use snell's law

$1 \times \sin i = \mu \times \sin i/2$

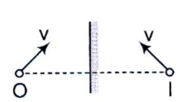
$2 \sin i/2 \times \cos i/2 = \mu \sin i/2$

15.

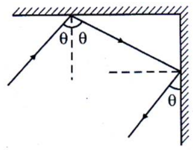
Angle between two velocities is 2θ.

$\therefore \quad v_{rel} = \sqrt{v^2 + v^2 - 2vv\sin 2\theta}$

$= 2v\sin\theta$



16.



The incident and the second reflected ray make the same angle θ with vertical. Therefore, they are parallel for any value of θ.

17.

$\sin\theta_1 = \frac{1}{\mu_g}$ and $\sin\theta_2 = \frac{1}{\mu_w}$

Since, $\mu_g > \mu_w$, $\theta_1 < \theta_2$

Critical angle θ between glass and water will be given by

$$\sin\theta = \frac{\mu_w}{\mu_g} \quad \text{or} \quad \theta > \theta_2$$

19. $\frac{1}{f} = \frac{1}{f'} + \frac{1}{f'}$

20. $P = P_1 + P_2$

21. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\frac{1}{f_1} = \left(\frac{\mu}{\mu/2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = \left(\frac{\mu}{2\mu} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

22. $f_l = f_a \frac{\left({}^a\mu_g - 1\right)}{\left({}^l\mu_g - 1\right)}$

23. $P = \frac{1}{f}, \ \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

25. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right), \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$

26. $d' = \frac{d}{\mu}$

27. Emergent ray will make an angle 60^0 with radial line at point of refraction.
Apply snell's law

29. From first lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$; $\frac{1}{v} - \frac{1}{-30} = \frac{1}{+15} \Rightarrow$

$\Rightarrow$ v = + 30 cm

Thus the first lens will form image at optical centre of lens L_2 and lens L_2 will form image at its optical itself because object distance is close to zero.

30. $\frac{1}{f_1} = \frac{1}{f_3} = (1.5 - 1)\left(-\frac{1}{20}\right)$

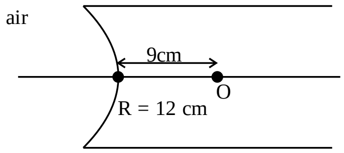
$$\frac{1}{f_2} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{20} - \frac{1}{-20}\right)$$

$$\frac{1}{F} = (0.5)\left(-\frac{1}{10}\right) + \left(\frac{1}{3}\right)\left(\frac{1}{10}\right) = -\frac{1}{20} + \frac{1}{30}$$

$$= \frac{-3 + 2}{60} \qquad \Rightarrow F = -60\,\text{cm}$$

31. Number of filling of refractive indices is 3 horizontally so number of images 3(3 focal lengths)

32.



$$\frac{1}{v}-\frac{1.5}{-9}=\frac{1-1.5}{+12}$$

$$\Rightarrow \frac{1}{v}+\frac{1.5}{9}=\frac{-0.5}{12}$$

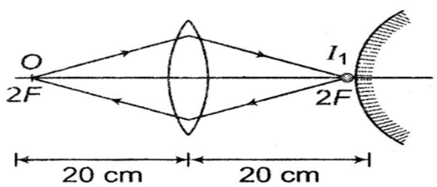
$$\Rightarrow \frac{1}{v}=-\frac{1}{6}-\frac{1}{24}$$

$$\Rightarrow \frac{1}{v}=\frac{-4-1}{24}$$

$$\Rightarrow v=-\frac{24}{5}$$

$$=-4.8\text{ cm}$$

33.



34. $\frac{\mu_2}{v}-\frac{\mu_1}{-2R}=\frac{\mu_2-\mu_1}{R}$

$v=\infty$

After reflection from the spherical surface the ray become parallel to optic axis. Hence after reflection from silvered surface it will retrace its initial path and image coincides with object.

$F=\frac{45}{7}cm$

36. Applying, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$

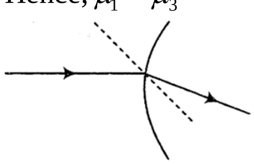
$\therefore \frac{1}{v}-\frac{3/2}{+30}=\frac{1-3/2}{+20}$

or $\frac{1}{v}=\frac{1}{20}-\frac{3}{40}$

$\therefore v=40\text{ cm}$

37. Between 1 and 3 there is no deviation.

Hence, $\mu_1=\mu_3$



Between 3 and 2

Ray to light binds towards normal. Hence,

$\mu_2>\mu_3$.

39. $\frac{3}{2}\times\sin 60=\mu$

40. $A=r_1+r_2$

$30^0 = r_1 + 0 \Rightarrow$ Using Snell's law

$$\frac{\sin i}{\sin r_1} = \mu$$

41. $c = \dfrac{x}{t_1}$

$v = \dfrac{10x}{t_2}$ & using $\sin c = \dfrac{1}{\mu}$

43. $${}_r\mu_d = \frac{1}{\sin\theta_c} = \frac{C_r}{C_d}$$

44. Let R be the radius of curvature of each surface. Then applying Lens Maker's Formula, we get

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{R}+\frac{1}{R}\right)$$

$\Rightarrow R = f$

For the water lens, again applying the Lens Maker's Formula, we get

$$\frac{1}{f'} = \left(\frac{4}{3}-1\right)\left(-\frac{1}{R}-\frac{1}{R}\right) = \frac{1}{3}\left(-\frac{2}{f}\right)$$

$$\Rightarrow \quad \frac{1}{f'} = -\frac{2}{3f}$$

Since, we know that for lenses placed in contact, we have

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f'}$$

$$\Rightarrow \quad \frac{1}{F} = \frac{2}{f} - \frac{2}{3f} = \frac{4}{3f}$$

$$\Rightarrow \quad F = \frac{3f}{4}$$

49. $\phi = \dfrac{\pi}{3}, a_1 = 4, a_2 = 3$

So, $A = \sqrt{a_1^2 + a_2^2 + 2a_1.a_2\cos\phi} \Rightarrow A \approx 6$

50. $$\beta = \frac{\lambda D}{d} \Rightarrow d = \frac{\lambda D}{\beta} = \frac{6000\times10^{-10}\times\left(40\times10^{-2}\right)}{0.012\times10^{-2}} = 0.2\,cm.$$

51. $d\sin\theta = (2n+1)\dfrac{\lambda}{2}$

$$d\left(\frac{y}{D}\right) = (2n+1)\frac{\lambda}{2}$$

here $D = d$ (given)
$d = b$ (given)

So,

$$\lambda = \frac{b^2}{d(2n+1)};$$

$(n = 0, 1, 2......)$

CHEMISTRY

15. $y_A P_T = P_A^O \times A$

$y_B P_T = P_B^O \times B$

$P_A^O > P_B^O$

$\Rightarrow \frac{y_A P_T}{X_A} > \frac{y_B P_T}{x_B}$

$\Rightarrow \frac{y_A}{y_B} > \frac{x_A}{x_B}$

17. Fact

20. Glucose is non electrolyte

22. $\Delta T_f = i k_f m$

24. PT= $P_A^O - \left(P_A^O - P_B^O\right) X_B$

27. Wt. of H_2SO_4 per litre $= N \times \text{eq. mass} = 2 \times 49 = 98g.$

28. In alkaline medium $KMnO_4$ act as oxidant as follows.

$2KMnO_4 + 2KOH \rightarrow 2K_2MnO_4 + H_2O + (O)$

Hence its $eq.wt. = m.wt. = 158$

Now, Normality $= \frac{\text{Mass}}{\text{Eq. mass}} \times \frac{1}{V_{(L)}}$

mass $= 0.1 \times 158 \times \frac{100}{1000} g = 1.58\ g.$

29. For complete neutralization equivalent of oxalic acid = equivalent of $NaOH$ =

$\frac{w}{eq.wt} = \frac{NV}{1000} \quad \therefore \frac{w}{63} = \frac{0.2 \times 100}{1000} \Rightarrow w = 1.26\ gm$.

30. F^- ions in $PPm = \frac{0.2}{500} \times 10^6 = 400$

31. 5.85 g $NaCl$ = 0.1 mol as it present in 1 kg of water ; molality $= \frac{wt.}{m\,wt. \times l} = \frac{5.85}{58.5 \times 1}$ =0.1$molal$

33. $\frac{P^o - P_s}{P^o} = \frac{n}{n+N}; \frac{P^o - P_s}{P^o} = \frac{w \times M}{m \times W} = 80$

34. $CH_3COCH_3 + CHCl_3$ is non ideal solution which shown negative deviation.

35. The particle come of $AlCl_3$ solution will be maximum due to ionisation less in $BaCl_2$ and minimum in urea

$AlCl_3 \rightarrow Al^{3+} + 3Cl^- = 4$

$BaCl_2 \rightarrow Ba^{2+} + 2Cl^- = 3$

More the number of particles in solution more is the osmotic pressure a colligative properties.

36. $\pi = \frac{5 \times 0.0821 \times 1000 \times 423}{342 \times 100} = 5.07\ atm$.

37. Osmotic pressure is a coligative properties equimolar solution of $Ca(NO_3)_2$ and Na_2SO_4 will produce same number of solute particles.

$CaNO_3 \rightleftharpoons Ca^{2+} + 2NO_3^-$

$Na_2(SO_4) \rightleftharpoons 2Na^+ + SO_4^{2-}$

38. $Al_2(SO_4)_3$ Deci-molar gives maximum ion. Hence, its osmotic pressure is maximum.

39. $NaCl$ and $ZnSO_4$ gives 2 ions but $NaCl$ is more ionic than $ZnSO_4$.

40. $m = \frac{K_b \times w \times 1000}{\Delta T_b \times W}$

$K_b = 2.16,\ w = 0.11,\ W = 15g,\ \Delta T_b = 0.1$

$m = \frac{2.16 \times 0.11 \times 1000}{0.1 \times 15} = 158.40 \simeq 158$.

41. The elevation (ΔT_b) in the boiling point

$= 354.11K - 353.23K = 0.88K$

Substituting these values in expression

$M_{\text{Solute}} = \frac{K_b \times 1000 \times w}{\Delta T_b \times W}$

Where, $w =$ weight of solute, $W =$ weight of solvent

$M_{\text{solute}} = \frac{2.53 \times 1.8 \times 1000}{0.88 \times 90} = 58\ gm\ mol^{-1}$

Hence, molar mass of the solute $= 58\ gm\ mol^{-1}$

42. $K_b = \frac{0.18}{0.1} = 1.8$

43. $m = \frac{K_f \times 1000 \times w}{W \times \Delta T_f} = \frac{5.1 \times 1000 \times 4.8}{60 \times 1.02} = 400$.

44. $\Delta T_f = mk_f$

$0.40 = \frac{0.01 \times 1000}{100} \times k_f \Rightarrow k_f = 4$

again $\Delta T_f = mk_f$

$= \frac{0.03 \times 1000}{50} \times 4$

$= 2.4$

45. $La(NO_3)_3$ will furnish four ions and thus will develop more lowering in freezing point whereas glucose gives only one particle and thus minimum lowering in freezing point.

46. $K_3[Fe(CN)_6] \rightarrow 3K^+ + [Fe(CN)_6]^{3-}$.

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